Sequential Design Method for Multivariable Decoupling and Multiloop PID Controllers

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Underlying algorithms for designing multivariable decoupling and multiloop PI/PID controllers in a sequential fashion are addressed. A single-loop technique, composed of biased relay identification schemes and tuning formulae leading to the minimum weighted integral of square error, is developed to tune each loop in the predetermined sequence of loop closing. The proposed tuning technique is appropriate for a wide range of process dynamics in a multivariable environment. A method is then proposed to design decouplers to compensate for the effect of interactions and tune the resultant weakly interacting, single-loop PI/PID controllers sequentially. The decouplers, together with the single-loop controllers, constitute the multivariable decoupling controller. If the interactions are not significant, multiloop PI/PID controllers, which do not incorporate decouplers, could be employed. Simulation and comparative results are shown for one 2 × 2 and one 3 × 3 multivariable system from the literature. Despite its simplicity, the proposed design method yields superior multivariable designs on the basis of performance, robust stability, and integrity.

1. Introduction

Any process capable of manufacturing or refining a product cannot operate satisfactorily within a single control loop. Virtually each unit operation requires at least two control loops to maintain the desired production rate and product quality (Shinskey, 1988). A large number of genuine multiloop control systems, which are made up of single-input/single-output (SISO) controllers acting in a multiloop fashion, have been reported (Vinante and Luyben, 1972; Wood and Berry, 1973; Ogunnaike and Ray, 1979; Tyreus, 1982). For such systems, loop interactions can arise and cause difficulties in feedback controller design. Cross-couplings of the process variables prevent the control engineer to design each loop independently. Adjusting controller parameters of one loop affects the performance of another, sometimes to the extent of destabilizing the entire system. To ensure stability, many industrial multiloop SISO controllers are tuned loosely, which causes inefficient operation and higher energy costs.

There are multiloop design methods that treat the multiloop system as an entity. Niederlinski (1971) proposed a heuristic method based on a generalization of the classical single-loop tuning method of Ziegler and Nichols (1942) to tune multiloop PID controllers. The method has not gained wide acceptance because of its complexity and some reports of poor performance (Waller, 1984). Luyben (1986) proposed the biggest log modulus tuning (BLT) method for multiloop PI controllers. The method first tunes each individual PI controller following the single-loop Ziegler−Nichols rules and then detunes the entire system by a single factor to meet a specific stability requirement. The method of Basualdo and Marchetti (1990) is a modification of the BLT method. First, the individual controllers are designed independently based on the internal model control (IMC) structure (Garcia and Morari, 1982). Then, a single parameter is employed to adjust the multiloop system until robust stability and performance conditions are satisfied. The latter two methods have the disadvantage of requiring excessive modeling effort to seek a complete transfer function matrix.

The idea of sequential design was used for multiloop control systems in recent years (O'Reilly and Leithead, 1991; Chiu and Arkun, 1992; Loh et al., 1993; Shen and Yu, 1994). According to specific sequential algorithms, the multivariable design problem is decomposed into a sequence of SISO design problems. Consequently, multiple single-loop designs can be employed by taking account of interactions in a sequential fashion. In this way, Loh et al. (1993) and Shen and Yu (1994) applied the single-loop relay technique of Åström and Hägglund (1984) to design multiloop PI controllers. However, the conventional relay technique based on the describing function stipulates that the output response resemble a sinusoidal wave, which is often not the case in the identification of multiloop systems in a sequential fashion.

There are also techniques for the design of true multivariable controllers that utilize all available process outputs jointly to make decisions on all inputs. With such controllers, it is possible to eliminate the effect of interactions between the process variables. Such techniques as the optimal control (LQ), the dynamic matrix control (DMC; Cutler and Ramaker, 1980), and the internal model control (IMC; Garcia and Morari, 1982) methods require the full knowledge of the process and the resultant controllers are often quite complex. Other approaches, such as Rosenbrock's inverse Nyquist array (INA) method, make use of interaction compensators (decouplers) to eliminate the interactions between the

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loops. The design algorithms are computationally involved and therefore less appealing to practicing engineers.

The objective of this work is to provide process engineers with an easy-to-use method to design multivariable decoupling and multiloop PI/PID controllers with very little prior knowledge about multivariable plants. The unknown multivariable plant is assumed square and open-loop stable. The design method must achieve good performance, robust stability, and integrity with minimal engineering effort.

Underlying algorithms are first presented to tune multiple PI/PID control loops in a multivariable plant. A single-loop tuning technique, composed of biased relay identification schemes and tuning formulae, is applied to each loop in the predetermined sequence of loop closing. The tuning technique is versatile for a wide range of process dynamics prevailing in a multivariable environment. Unlike the conventional relay technique, the proposed identification schemes require only the existence of sustained oscillations and always lead to exact estimates of the frequency responses at zero and relay frequencies.

If the multivariable system experiences severe interactions, a method is proposed to design decouplers to compensate for the effect of interactions. Otherwise, multiloop PI/PID controllers, which do not incorporate decouplers, could be employed. The decouplers are constructed based on approximate models obtained from proposed biased relay tests. Tuning of the resultant weakly interacting, single-loop PI/PID controllers is then achieved in the predetermined sequence of loop closing. The decouplers, together with single-loop controllers, constitute the multivariable decoupling controller.

The proposed design method only assumes that pairing of the manipulated and controlled variables has been made using measures such as the Niederlinski index (Niederlinski, 1971; Grosdidier, et al., 1985) so that the system is PI/PID stabilizable. Simulation results are shown for one 2 × 2 and one 3 × 3 multivariable systems from the literature. The proposed design method compares favorably with the Niederlinski, BLT, and empirical methods.

2. Sequential Tuning of Multiple Control Loops

A multivariable decoupling controller consists of n single-loop PID controllers together with $n!$ decouplers. The $n \times n$ process $G_P(s)$ is represented as follows:

$$G_P(s) = [g_{ij}(s)] \qquad i, j = 1, 2, ..., n \tag{1}$$

The centralized control structure is better illustrated with a 2 × 2 system in Figure 1. Additional transfer function blocks (decouplers) can be introduced between the single-loop controllers and the process. The main objective in decoupling is to compensate for the effect of loop interactions brought about by cross-couplings of the process variables. The design of decouplers will be elaborated in a later section. If the decouplers are absent, the control structure shown in Figure 1 reduces to a multiloop control system.

The variables $r_i(s)$, $u_i(s)$, $v_i(s)$, and $y_i(s)$ represent, respectively, the reference (setpoint), input, manipulated, and output (controlled) variables of loop i. As depicted in Figure 1, the PID controller of loop i is in

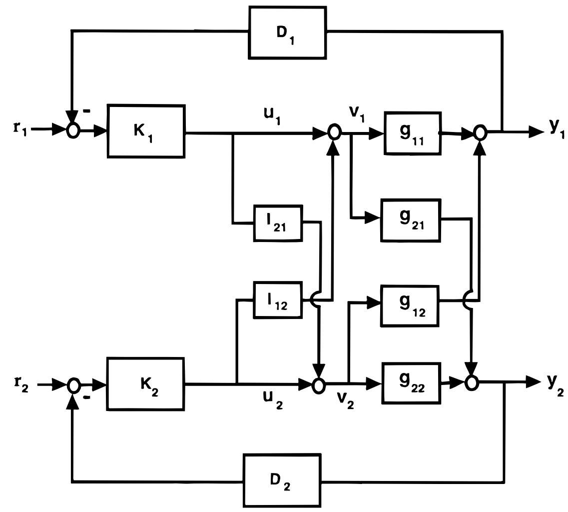


Figure 1. Double-loop decoupling PID control system.

the form of series connection, which consists of the following:

$$K_i(s) = k_{Ci}\left(1 + \frac{1}{\tau_{Ii}s}\right) \tag{2a}$$

$$D_i = \frac{1 + \tau_{Di}s}{1 + \alpha\tau_{Di}s} \tag{2b}$$

This series connection of the proportional-integral action and the derivative action is utilized to avoid a derivative kick for an abrupt change in the reference input. The term $(1 + \alpha\tau_{Di}s)$ is added to render the controller physically realizable. The value of α is typically between 0.05 and 0.2.

Even if the decouplers are incorporated, interactions between the process variables cannot be completely eliminated because of model mismatch. As a result, the multiple control loops for a multivariable process even with decouplers can not, in general, be tuned independently. Here, a sequential tuning strategy that takes interactions into consideration is proposed. The underlying idea is to treat multiple control loops with or without decouplers as a sequence of single loops. For example, the two control loops in the 2 × 2 system can be tuned in a sequential way of loop closing as depicted in Figure 2. Loop 1 is first tuned with loop 2 open (see Figure 2a) by performing a closed-loop identification test on loop 1 to determine the corresponding controller settings. Loop 1 is then placed in automatic with the resultant settings and an identification test is performed on loop 2 (see Figure 2b). With loop 2 closed, an identification test may be performed again on loop 1 to provide a new set of controller settings (see Figure 2c). This sequential tuning procedure can be continued between Figures 2b and 2c until the convergence of all controller parameters is achieved. Note that for each tuning stage, the system will behave like a single loop. The double-loop results can be extended readily to the case of n coupled control loops to be tuned sequentially.

It is then clear that multiple control loops can be tuned sequentially in an iterative manner using any single-loop tuning technique. Two questions that arise are: In what sequence should multiple control loops be tuned?; and Why is one sequence advantageous over another? These questions must be answered by examining the mutual effect between the loops via interac-

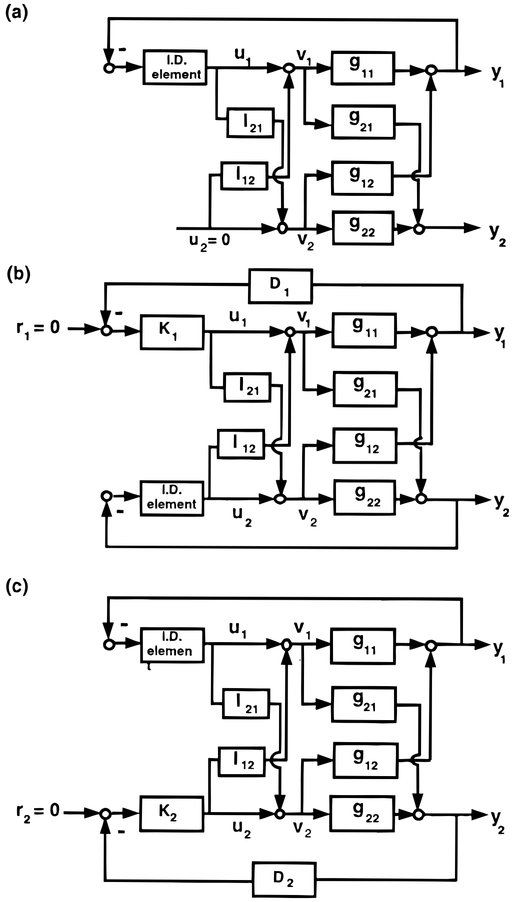


Figure 2. Sequential tuning procedure illustrated with a double-loop control system.

tions. It has been reported several times that in a multiloop control system, a faster loop is less affected via interactions with a slower loop but not vice versa (McAvoy, 1983; Isermann, 1991; Hwang, 1995a). Loh et al. (1993) and Hwang (1995a) further indicated that the speed of a control loop can be estimated roughly by the ultimate frequency of the freely standing loop (i.e., the diagonal element of the process transfer matrix alone). The ultimate frequency is referred to as the frequency of sustained oscillations resulting from a purely proportional control loop. Such considerations result in the rule of thumb that the tuning sequence should be started with the faster loops with higher ultimate frequencies. This sequence allows the slower loops to be tuned later and, hence, enables one to account for the interactions resulting from the closure of the faster loops (Loh et al., 1993). A second rule is that a loop that is significantly faster than some other loops can be treated as a decoupled loop and tuned independently regardless of variations of the controller settings in these slower loops. In this way, the number of iterative tuning steps can be kept to a minimum.

Based on the aforementioned arguments and extensive simulation study, a clear and efficient sequential tuning procedure is developed for multiple control loops as follows:

Multivariable Decoupling Controller. (1) Without the incorporation of decouplers, perform prior tests on each individual loop (freely standing with all other loops open) to estimate the ultimate frequency. Rank the loop speeds from fast to slow based on the estimated ultimate frequencies.

(2) With the decouplers added, the controller design task is reduced to tuning of several weakly interacting, single-loop controllers, as depicted in Figure 2. The tuning strategy is then to tune the loops once in the fast-to-slow sequence of loop closing. Further iterations are not required because the loop interactions are weak.

Multiloop Controllers. (1) Perform prior tests on each individual loop to estimate the ultimate frequency. Rank the loop speeds from fast to slow based on the estimated ultimate frequencies.

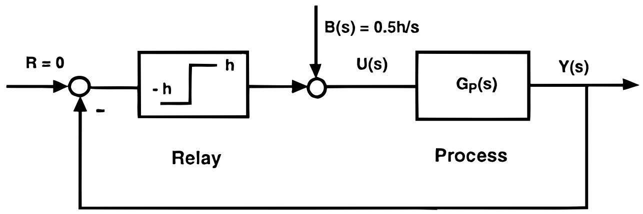
(2) For a multiloop system, multiple control loops with rather significant interactions may be encountered. The tuning strategy is more elaborate. First, decompose the multiloop system into several subsystems by looking through the estimated ultimate frequencies. Then, let all loops in a more rapid subsystem have ultimate frequencies at least twice those in a slower subsystem. Loops of comparable speeds are grouped into the same subsystem (the ratio of any two adjacent ultimate frequencies is <2). It is assumed that a slower subsystem has a negligible effect on a faster one via loop interactions but not vice versa. For example, in a four-loop system, the ultimate frequencies of the individual loops are estimated to be 3, 2, 1, and 0.75. The system can be split into two subsystems, a faster subsystem consisting of the first two loops and a slower subsystem consisting of the latter two loops. In this way, a higher dimensional problem can be decomposed into several lower dimensional problems if some of the ultimate frequencies of the individual loops are distinct and far apart.

(3) Tune the subsystems in the fast-to-slow sequence of loop closing. The fastest subsystem is first tuned with all slower subsystems open. It is then placed in automatic and the second fast subsystem is tuned with the rest of the subsystems open. This sequence is continued until the slowest subsystem is tuned. Note that because the closure of the slower subsystem has a negligible effect on the faster subsystem via interactions, there is no need to retune the faster subsystem after the slower subsystem is closed. In other words, each subsystem is tuned independently in the fast-to-slow sequence.

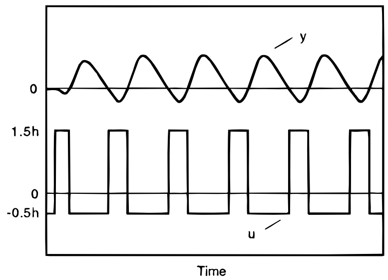
(4) When a subsystem consisting of more than one loop is to be tuned, the loops should also be tuned in the fast-to-slow sequence of loop closing. However, these loops are now of comparable speeds, implying that they are coupled with one another to a certain extent. Therefore, the tuning sequence should be repeated in an iterative manner to account for the effect of loop interactions. For example, a two-loop subsystem (loop 1 is faster than loop 2) should be tuned in the repeated sequence of 1−2−1−2, as illustrated in Figure 2. Extensive simulation study reveals that further iterations do not necessarily improve the performance.

3. Single-Loop Tuning Technique for Multivariable Systems

By virtue of the sequential tuning algorithms, the design of a multivariable control system reduces to multiple single-loop designs. In principle, any single-loop PID tuning method can be applied. However, during the sequential tuning procedure, the combined dynamics of a single loop could exhibit widely different characteristics from those of the freely standing loops (at least higher order than the original). Such difference

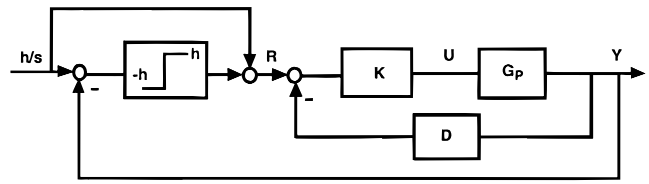


(a) Biased relay structure

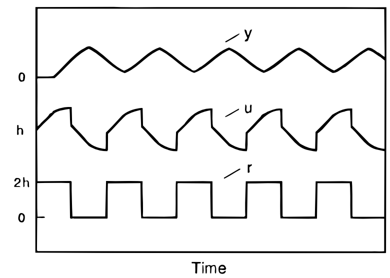


(b) Typical input-output responses

Figure 3. Biased relay identification scheme.



(a) Cascade relay structure



(b) Typical input-output responses

Figure 4. Cascade relay identification scheme.

is caused by the closure of a subset of loops and/or the incorporation of decouplers. The combined dynamics could be oscillatory, nonminimum phase, or high order even if the dynamics of all elements in the transfer matrix are overdamped, minimum phase, and low order. In other words, a practical single-loop tuning method, if applicable to multivariable systems, must be able to cope with a very wide variety of dynamics. The well-known Ziegler−Nichols PID tuning formulae, even modified, are not suitable.

Here, a single-loop tuning technique is developed to tune multiple PID control loops sequentially. The technique is a combination of relay-based identification schemes and PI/PID tuning formulas. For identification purposes, the conventional relay feedback scheme of Åström and Hägglund (1984) has the merits that *a priori* information about the time scale or the structure of the process is not needed and that a rapid excitation signal is generated automatically. In this work, the conventional relay scheme is adapted to suit the identification of sophisticated multivariable dynamics.

3.1. Biased Relay Identification Scheme. The SISO process is excited by connecting a relay in a feedback path from the process output to the input, as depicted in Figure 3a. The relay has an amplitude h. A bias $B(s)$ ($= 0.5h/s$) is added at the process input to change the relay characteristics, producing a nonzero-mean square signal. The process input and output signals would become limit cycles at equilibrium. Because of the bias added, the limit cycles, $u(t)$ and $y(t)$, are asymmetrical in the relay switching intervals, as shown in Figure 3b. These limit cycles in the process input and output can be used to obtain a model for subsequent controller tuning.

3.2. Cascade Relay Identification Scheme. Biased relay identification can also be performed on the process under closed-loop control provided that stabilizing controller settings are available (Schei, 1992; Hwang, 1995b). This identification would lead to a process model that describes better the dynamics under normal operating conditions. As shown in Figure 4a, the PID control system is excited by connecting a relay in a feedback path from the process output to the controller reference. With a step change of magnitude h applied before and after the relay, the reference signal $r(t)$ will vary in a square form between 0 and 2, as illustrated in Figures 4a and 4b, where the amplitude of the relay function is also h. Typical process input and output limit cycles, $u(t)$ and $y(t)$, are also plotted in Figure 4b.

3.3. Derivation of Process Model from a Single Biased Relay Test. The conventional relay feedback test without bias can only yield the estimates of the ultimate gain and frequency, which are not sufficient to obtain an appropriate process model. Hence, the Ziegler−Nichols rules must be used for subsequent controller tuning. On the other hand, the proposed biased relay schemes in conjunction with the estimation algorithm discussed next can be used to estimate accurately the steady-state gain and one point on the Nyquist curve, which leads to a process model with three parameters.

The process transfer function is given as follows:

$$g(s) = \frac{Y(s)}{U(s)} \tag{3}$$

Signals $y(t)$ and $u(t)$ in both identification structures will oscillate perpetually (limit cycles) with the same frequency ω_o, the relay frequency, after the transients die out. Replacing s by $j\omega_o$ and applying the definition of Laplace transform in eq 3 yields eq 4:

$$g(j\omega_o) = \frac{\int_0^\infty y(t)\cos\omega_o t\,dt - j\int_0^\infty y(t)\sin\omega_o t\,dt}{\int_0^\infty u(t)\cos\omega_o t\,dt - j\int_0^\infty u(t)\sin\omega_o t\,dt} \tag{4}$$

Because of the periodicity of the signals, the evaluation of the integrals in eq 4 can be greatly simplified as integrating over any number of periods; that is

$$g(j\omega_o) = \frac{\int_{t_0}^{t_0+mT} y(t)\cos\omega_o t\,dt - j\int_{t_0}^{t_0+mT} y(t)\sin\omega_o t\,dt}{\int_{t_0}^{t_0+mT} u(t)\cos\omega_o t\,dt - j\int_{t_0}^{t_0+mT} u(t)\sin\omega_o t\,dt} \tag{5}$$

where t_0 denotes an arbitrary time after the transients die out, m is an integer, and T (= $2\pi/\omega_o$) is the period.

The steady-state gain k_P is estimated as follows:

$$k_P = g(0) = \frac{\int_{t_0}^{t_0+mT} y(t)\,dt}{\int_{t_0}^{t_0+mT} u(t)\,dt} \tag{6}$$

Note that the conventional relay technique, generating zero-mean limit cycles at the process input and output, is not capable of exploiting this relation because both of the integrals in eq 6 are zeros.

Equation 5 constitutes the basis of estimating one point on the Nyquist curve of the process (i.e., the amplitude ratio and phase angle of the frequency response at the relay frequency ω_o). As is well known from the Fourier series analysis for periodic signals, ω_o is essentially the frequency of the fundamental harmonic. The integrals in eq 5 can eliminate the components of means and higher harmonics in y and u. Equation 5 thus leads to the exact estimate of one point on the Nyquist curve of the process. This ability is a distinct advantage of the proposed relay schemes over the conventional one, which simply ignores the effect of higher harmonics.

The proposed schemes in conjunction with eqs 5 and 6 require only the existence of sustained oscillations regardless of the shape of limit cycles. On the contrary, the technique based on the describing function stipulates that the output response resemble a sinusoidal wave. Furthermore, the proposed schemes applying a bias at the relay output easily yield the estimates of the steady-state gain as well as one Nyquist point from a single test. These features are particularly suited for identification task of multivariable systems in a sequential fashion. Another advantage of the proposed relay schemes is their insensitivity towards measurement noise and static disturbances. The algorithm (eq 5) can eliminate essentially any static upsets arising in the process input or output, except that the oscillation frequency might be slightly altered. The effects of measurement noise during identification are attenuated by the feature of integrating over cycles.

For identification purposes, a first-order-plus-time-delay model is employed:

$$g(s) = \frac{k_P e^{-ds}}{\tau s + 1}$$

A wide range of d/τ is assumed to account for a wide variety of process dynamics. The model can simulate high-order, delayed, slightly oscillatory, and small non-minimum phase dynamics. The estimates of the steady-state gain and one Nyquist point obtained from a single relay test directly yield the three model parameters.

3.4. PID Controller Settings for First-Order-Plus-Time-Delay Processes. To account for the complexity of the combined dynamics in a multivariable system, a wide range of d/τ ($0.01 \le d/\tau \le 10$) is considered in deriving PI/PID parameters. The ideal PID controller is given in one of the following forms:

$$g_C(s) = k'_C\left(1 + \frac{1}{\tau'_I s} + \tau'_D s\right)$$
$$= k_C\left(1 + \frac{1}{\tau_I s}\right)(1 + \tau_D s) \tag{7}$$

The relation between the parameters of series connection, k_C, τ_I, and τ_D, and those of parallel connection, k'_C, τ'_I, and τ'_D, is

$$k_C = k'_C(1 + \sqrt{1 - 4\tau'_D/\tau'_I})/2$$
$$\tau_I = \tau'_I(1 + \sqrt{1 - 4\tau'_D/\tau'_I})/2$$
$$\tau_D = \tau'_I(1 - \sqrt{1 - 4\tau'_D/\tau'_I})/2 \tag{8}$$

Although PID parameters of series connection are sought in the present work, the structure of parallel connection is assumed in the derivation of PID parameters for convenience. Equation 8 is used to convert PID parameters of parallel connection to those of series connection defined in eq 2. The optimization of PI and PID control is primarily based on the performance index, the weighted integral of the square error (WISE), proposed by Nishikawa et al. (1984). We intend to minimize the WISE of the closed-loop response subject to a step change in setpoint:

$$\text{WISE}(\beta) = \int_0^\infty [\Delta y(t) e^{\beta t}]^2\,dt \tag{9}$$

where $\Delta y(t) = r(t) - y(t)$ is the error of the process output and the positive number β is to adjust the damping of the response. The optimal controller settings that minimize the index (eq 9), with β chosen appropriately, yield fast damping and small overshoot characteristics. The parameter β is set by

$$\beta = \frac{\gamma}{P_u} \tag{10}$$

where P_u is the ultimate period of the process. The value of γ is chosen so that the integral of the absolute error (IAE) of the response is minimal. For PID control, the ratio of τ'_D to τ'_I is specified to simplify the search procedure without sacrificing the of control system performance.

The optimal PI and PID parameters that minimize WISE with appropriate γ are searched after by an optimization routine for a wide range of d/τ (i.e., $0.25 \le d/\tau \le 10$). For very small d/τ (e.g., $d/\tau < 0.25$), simulation results show that the WISE method tends to yield aggressive controller settings, producing large overshoot and poor robustness. For this range of d/τ, we choose to use the IMC−PID design proposed by Rivera et al. (1986), which leads directly to a PI or PID controller. However, because of the substitution of a Padé approximation for the delay term, the IMC−PID design is inferior to the WISE method for large d/τ. The resultant tuning formulas are

PI control ($0.01 \le d/\tau \le 10$)

$$k_C k_P = \begin{cases} \tau/(0.005\tau + 1.53d) & \text{for } d/\tau < 0.25 \\ 0.292 + 0.482(\tau/d) + 0.023(\tau/d)^2 & \text{for } d/\tau \ge 0.25 \end{cases} \quad (11a)$$

$$\tau_I/\tau = \begin{cases} 1 & \text{for } d/\tau < 0.25 \\ 0.955 + 0.386(d/\tau) & \text{for } d/\tau \ge 0.25 \end{cases} \quad (11b)$$

PID control ($0.01 \le d/\tau \le 10$)

$$k'_C k_P = \begin{cases} (\tau + 0.5d)/(0.005\tau + 1.2d) & \text{for } d/\tau < 0.25 \\ 0.374 + 0.724(\tau/d) + 0.025(\tau/d)^2 & \text{for } d/\tau \ge 0.25 \end{cases} \quad (12a)$$

$$\tau'_I/\tau = \begin{cases} 1 + 0.5(d/\tau) & \text{for } d/\tau < 0.25 \\ 0.966 + 0.482(d/\tau) & \text{for } d/\tau \ge 0.25 \end{cases} \quad (12b)$$

$$\tau'_D/\tau = \begin{cases} d/(2\tau + d) & \text{for } d/\tau < 0.25 \\ [0.5d/(\tau + 0.5d)^2]\tau'_I & \text{for } d/\tau \ge 0.25 \end{cases} \quad (12c)$$

Note that the aforementioned functions are the combination of the WISE method for $d/\tau \ge 0.25$ and IMC method for $d/\tau < 0.25$. It is important that the functions are continuous at the point of $d/\tau = 0.25$ so that iterations over loops using the formulae can proceed in the neighborhood of this point. To meet this requirement, the filter time constant τ_C in the IMC design is calculated to be $0.005\tau + 0.53d$ for PI control and $0.005\tau + 0.7d$ for PID control, whereas the ratio of τ'_D and τ'_I derived from the IMC design is also used in the WISE method.

To demonstrate the validity of the aforementioned tuning formulae, eq 11 is compared with the IMC−PI method and the Ziegler−Nichols PI method in view of performance, overshoot, and robustness for first-order-plus-time-delay processes in Figure 5. The filter time constant of the IMC−PI rules is selected to be $0.7d$ so that the resultant control system yields the minimum IAE for a step setpoint change. It appears that for a wide range of d/τ, the proposed formulae yield satisfactory IAE for setpoint and load changes. They also provide robust stability and small overshoot. Robust stability is measured via the index proposed by Doyle and Stein (1981); that is, the lower bound of the singular value of $1 + (g\, g_C)^{-1}$ for all frequencies. On the other hand, the Ziegler−Nichols and IMC methods are valid only for a relatively narrow range of d/τ. For example, both methods lead to poor performance for large d/τ, as seen in Figures 5a and 5b. The Ziegler−Nichols method produces excessively large overshoot and provides poor robust stability for small d/τ, as shown in Figure 5c.

4. The Design of Decouplers

In multiloop design, the underlying theme is not to eliminate interactions entering the loops but to take them into account loop by loop in a sequential fashion. In other words, an optimally tuned multiloop control

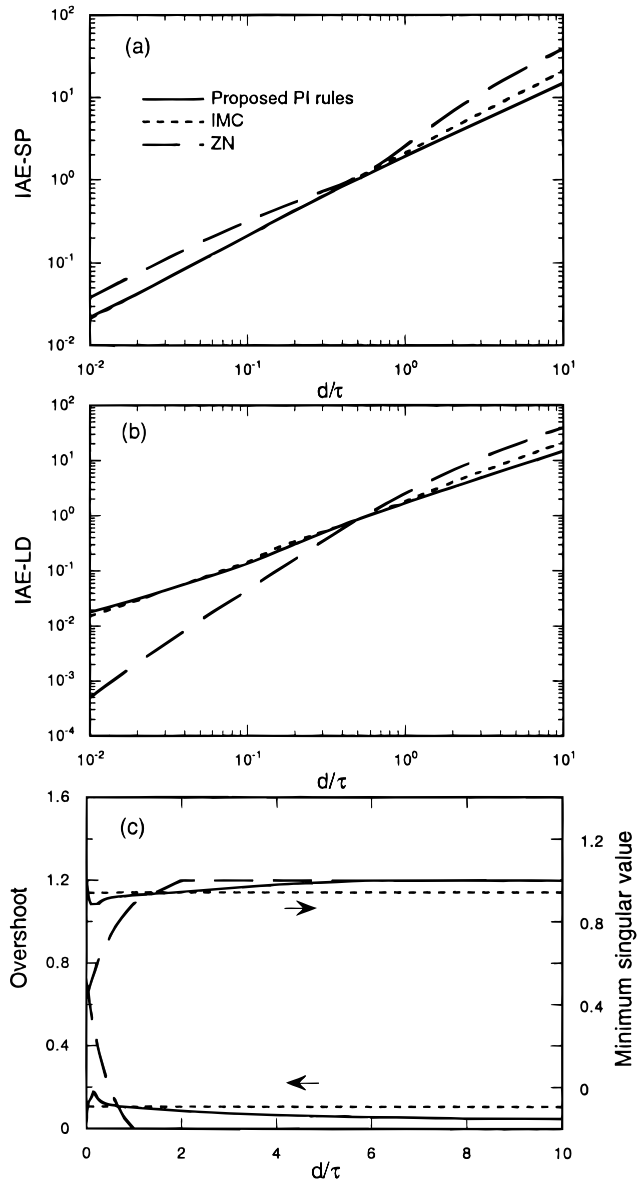


Figure 5. Comparison of the proposed PI tuning rules with the Ziegler−Nichols PI and IMC−PI rules via (a) IAE values for step setpoint changes, (b) IAE values for step load changes, and (c) overshoot and robustness index.

system may still experience severe loop interactions and respond poorly. On the other hand, the objective in decoupling is to compensate for the effect of loop interactions brought about by cross-couplings of the process variables.

The decoupler design problem is to eliminate interactions from all loops. We start with the decoupler design for a 2×2 system shown in Figure 1, where the process model is

$$y_1 = g_{11}\nu_1 + g_{12}\nu_2 \quad (13a)$$

$$y_2 = g_{21}\nu_1 + g_{22}\nu_2 \quad (13b)$$

Because of decouplers, the equations governing the control action become

$$\nu_1 = u_1 + I_{12}u_2 \quad (14a)$$

$$\nu_2 = u_2 + I_{21}u_1 \quad (14b)$$

Substituting these equations into eq 13 gives

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} g_{11} + g_{12}I_{21} & g_{11}I_{12} + g_{12} \\ g_{21} + g_{22}I_{21} & g_{21}I_{12} + g_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (15)$$

Now in order to have u_1 affect only y_1 and u_2 affect only y_2, we choose the decouplers as follows:

$$I_{12} = -\frac{g_{12}}{g_{11}} \quad (16a)$$

$$I_{21} = -\frac{g_{21}}{g_{22}} \quad (16b)$$

Now, the two loops appear to act independently and can therefore be tuned individually in isolation from the other.

Perfect decoupling is only possible if the exact process model is available. As this is hardly ever the case, perfect decoupling is unattainable in practice. Nevertheless, imperfect decouplers, based on the identified approximate models, can be implemented easily as follows. Prior to the incorporation of decouplers, the proposed biased relay scheme is applied to each individual loop with the other loop open. Each element g_{ij} is identified as a first-order-plus-time-delay transfer function:

$$g_{ij}(s) = \frac{k_{ij}e^{-d_{ij}s}}{\tau_{ij}s + 1} \quad (17)$$

Note that each identification test yields not only a diagonal element but also an off-diagonal element. For example, for the identification on loop 1 with loop 2 open, the diagonal element g_{11} is obtained from the excited u_1–y_1 limit cycles; simultaneously, the off-diagonal element g_{21} is estimated from the excited u_1–y_2 limit cycles. Hence, for an $n \times n$ system, merely n such tests are required to construct decouplers. The decouplers then become

$$I_{12}(s) = -\frac{k_{12}(\tau_{11}s + 1)}{k_{11}(\tau_{12}s + 1)}e^{-(d_{12}-d_{11})s} \quad (18a)$$

$$I_{21}(s) = -\frac{k_{21}(\tau_{22}s + 1)}{k_{22}(\tau_{21}s + 1)}e^{-(d_{21}-d_{22})s} \quad (18b)$$

The decouplers must be stable and causal. Stability is ensured because the identification schemes lead to a stable transfer function without RHP zeros. To ensure causality, it is necessary that the smallest time delay occurs on the diagonal (i.e., $d_{11} < d_{12}$ and $d_{22} < d_{21}$). If this is not the case, an approximation of the predictor based on the Taylor series expansion is used:

$$e^{+Ls} \approx \frac{Ls + 1}{0.2Ls + 1} \quad (19)$$

For a 3 × 3 system, there are six decouplers to be implemented. The resultant relationship between the process outputs y_i and the outputs of the single-loop controllers u_i becomes

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} g_{11} + g_{12}I_{21} + g_{13}I_{31} & g_{11}I_{12} + g_{12} + g_{13}I_{32} & g_{11}I_{13} + g_{12}I_{23} + g_{13} \\ g_{21} + g_{22}I_{21} + g_{23}I_{31} & g_{21}I_{12} + g_{22} + g_{23}I_{32} & g_{21}I_{13} + g_{22}I_{23} + g_{23} \\ g_{31} + g_{32}I_{21} + g_{33}I_{31} & g_{31}I_{12} + g_{32} + g_{33}I_{32} & g_{31}I_{13} + g_{32}I_{23} + g_{33} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \quad (20)$$

Complete decoupling requires that the six off-diagonal elements of the matrix in eq 20 be set to zeros. Every two decouplers are obtained by solving two simultaneous equations in the same column. For example, the decouplers for loop 1 are obtained as

$$I_{1m} = \frac{g_{nm}g_{1n} - g_{1m}g_{nn}}{g_{11}g_{nn} - g_{1n}g_{n1}} = \frac{g_{nm}g_{1n}}{g_{nn}(g_{11} - g_{1n}g_{n1}/g_{nn})} - \frac{g_{1m}g_{nn}}{g_{11}(g_{nn} - g_{1n}g_{n1}/g_{11})} \quad m \neq n;\ m, n = 2, 3 \quad (21)$$

With three biased relay tests performed on the individual loops, each element g_{ij} is identified as a first-order-plus-time-delay transfer function. For the ease of implementation, each combined term in the parentheses of eq 21 is further simplified to a first-order-plus-time-delay model by letting their steady-state gains and frequency responses at a properly chosen frequency be identical; that is

$$g_{11} - \frac{g_{1m}g_{m1}}{g_{mm}} \approx \frac{k'_{1m}e^{-d'_{1m}s}}{\tau'_{1m}s + 1}$$

$$g_{mm} - \frac{g_{1m}g_{m1}}{g_{11}} \approx \frac{k'_{mm}e^{-d'_{mm}s}}{\tau'_{mm}s + 1} \quad m = 2, 3 \quad (22)$$

This frequency for fitting can be selected as the slowest of the three relay frequencies in the biased relay tests, implying that the control system is decoupled at least for the dominant frequency of the slowest loop. It should be mentioned that although the decouplers are sometimes constructed at the smallest frequency, the entire system is not designed only for that frequency. In a sequential fashion, the single-loop controllers are tuned to reflect the speed of each loop. Substituting the models in eq 22 into eq 21 leads to the simplified decouplers for loop l:

$$I_{lm} = \frac{k_{nm}k_{ln}(\tau_{nn}s + 1)(\tau'_{ln}s + 1)}{k_{nn}k'_{ln}(\tau_{nm}s + 1)(\tau_{ln}s + 1)}e^{-(d_{nm}+d_{ln}-d_{nn}-d'_{ln})s} - \frac{k_{lm}k_{nn}(\tau_{ll}s + 1)(\tau'_{nn}s + 1)}{k_{ll}k'_{nn}(\tau_{lm}s + 1)(\tau_{nn}s + 1)}e^{-(d_{lm}+d_{nn}-d_{ll}-d'_{nn})s} \quad l \neq m \neq n;\ l, m, n = 1, 2, 3 \quad (23)$$

If any delay term in eq 23 is not causal, the approxima-

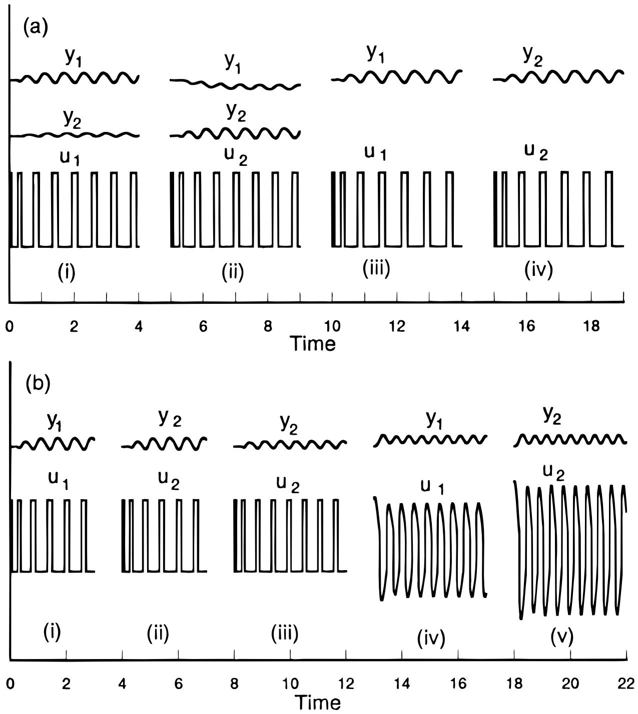


Figure 6. Tuning procedures including prior tests for example 1 with the first pairing: (a) multivariable decoupling PID design; (b) multiloop PID design.

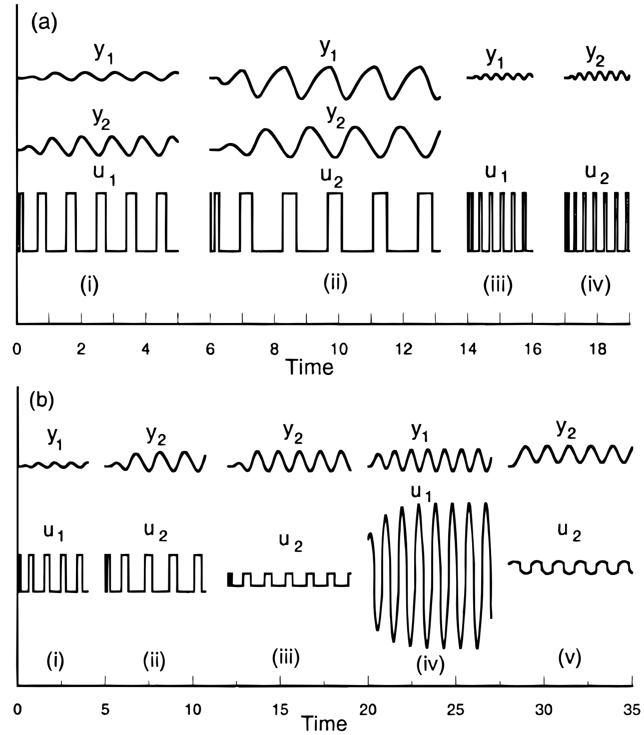


Figure 7. Tuning procedures including prior tests for example 1 with the second pairing: (a) multivariable decoupling PID design; (b) multiloop PID design.

tion (eq 19) could be used. Such decoupler design can be easily extended to $n \times n$ systems.

For 3×3 or higher dimensional systems, one may consider partial decoupling when some of the loop interactions are weak or if some of the loops need not have high performance. Therefore, attention is focused on a subset of decouplers such that only a portion of the six off-diagonal elements of the matrix in eq 20 are set to zeros.

5. Multivariable Cases Studied

The proposed sequential design method for multivariable decoupling and multiloop PI/PID controllers are applied to one 2×2 and one 3×3 system from the literature.

Example 1 (Niederlinski, 1971).

$$G_{P1}(s) = \begin{bmatrix} \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{-2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \\ \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{1}{(0.1s+1)(0.2s+1)^2} \end{bmatrix}$$

$$G_{P2}(s) = \begin{bmatrix} \dfrac{0.5}{(0.1s+1)^2(0.2s+1)^2} & \dfrac{-1}{(0.1s+1)(0.2s+1)^2} \\ \dfrac{1}{(0.1s+1)(0.2s+1)^2} & \dfrac{2.4}{(0.1s+1)(0.2s+1)^2(0.5s+1)} \end{bmatrix}$$

For this example, two possible pairing configurations, reflected by the transfer function matrices G_{P1} and G_{P2}, are considered. Both pairings are integral stabilizable. For the first pairing, decouplers could be added to improve performance by diminishing loop interactions. The sequential design procedure for a multivariable decoupling PID controller is shown in Figure 6a and delineated as follows:

Step 1. Without the incorporation of decouplers, perform a biased relay test on loop 1 with loop 2 open. The input u_1 and the outputs y_1 and y_2 are recorded and the first-order-plus-time-delay models for g_{11} and g_{21} are derived based on eqs 5 and 6. The ultimate frequency of loop 1 is estimated to be 10.9 based on the model.

Step 2. Perform a biased relay test on loop 2 with loop 1 open. The input u_2 and the outputs y_1 and y_2. The ultimate frequency of loop 2 is estimated to be 10.9. The decouplers are then designed using eq 18 as follows:

$$I_{12} = \frac{2.4(0.7s+1)e^{-0.122s}}{3.73s+1}$$

$$I_{21} = -\frac{0.5(0.7s+1)e^{-0.0735s}}{1.01s+1}$$

Install them in place. Because the two loops have the same ultimate frequencies, the tuning sequence can be 1−2 or 2−1. Select arbitrarily the tuning sequence as 1−2.

Step 3. Loop 1 is tuned using a biased relay with loop 2 open.

Step 4. Loop 2 is then tuned using a biased relay with loop 1 closed with the PID settings obtained in step 3.

The sequential tuning procedure for multiloop PID controllers is shown in Figure 6b and delineated as follows:

Steps 1 and 2. Perform a biased relay test on each individual loop with the other loop open as for the multivariable decoupling controller. The tuning sequence can be 1−2−1−2 or 2−1−2−1. Select arbitrarily the sequence of 1−2−1−2.

Step 3. Loop 2 is then tuned using a biased relay with loop 1 closed with the PID settings obtained in step 1.

Step 4. Loop 1 is retuned using the cascade relay scheme with loop 2 closed. The PID parameters used for loop 2 are obtained in step 3 and the PID controller placed in loop 1 remains unchanged for identification.

Step 5. Loop 2 is retuned using the cascade relay scheme with loop 1 closed.

Note that the multiloop system is characterized by two comparable ultimate frequencies (both are 10.9); some iterations are required to achieve better design.

For the second pairing, the sequential design procedure for a multivariable decoupling PID controller is shown in Figure 7a. The ultimate frequencies of loops 1 and 2 are estimated to be 7.07 and 4.74, respectively. The decouplers are designed using eqs 18 and 19 as follows:

$$I_{12} = \frac{2(0.587s + 1)(0.0676s + 1)}{(0.379s + 1)(0.2 \times 0.0676s + 1)}$$

$$I_{21} = -\frac{0.417(1.06s + 1)(0.197s + 1)}{(0.48s + 1)(0.2 \times 0.197s + 1)}$$

Note that the original decouplers based on eq 18 cannot be built and, hence, eq 19 is employed to yield the approximate decouplers. With the decouplers installed, the system is tuned in the sequence of 1−2. The sequential tuning procedure for multiloop PID controllers are shown in Figure 7b. The tuning sequence is 1−2−1−2.

Example 2 (Tyreus, 1982).

$$G_P(s) = \begin{bmatrix} \frac{1.986e^{-0.71s}}{66.7s + 1} & \frac{-5.24e^{-60s}}{400s + 1} & \frac{-5.984e^{-2.24s}}{14.29s + 1} \\ \frac{-0.0204e^{-0.59s}}{(7.14s + 1)^2} & \frac{0.33e^{-0.68s}}{(2.38s + 1)^2} & \frac{-2.38e^{-0.42s}}{(1.43s + 1)^2} \\ \frac{-0.374e^{-7.75s}}{22.22s + 1} & \frac{11.3e^{-3.79s}}{(21.74s + 1)^2} & \frac{9.811e^{-1.59s}}{11.36s + 1} \end{bmatrix}$$

For the Tyreus distillation column, the ultimate frequencies of loops 1−3 are estimated to be 2.22, 1.06, and 1.04, respectively. The decouplers for complete decoupling are designed as follows:

$$I_{21} = \frac{0.0068(6.41s + 1)(156s + 1)(0.487s + 1)}{(86.4s + 1)(11.2s + 1)(0.2 \times 0.487s + 1)} + \frac{0.0314(11.2s + 1)(90.5s + 1)e^{-5.47s}}{(23.2s + 1)(2.61s + 1)}$$

$$I_{31} = \frac{0.0044(11.2s + 1)(90.5s + 1)e^{-5.95s}}{(6.41s + 1)(23.2s + 1)} - \frac{0.0078(6.41s + 1)(156s + 1)e^{-3.24s}}{(86.4s + 1)(474s + 1)}$$

$$I_{12} = \frac{3.02(66.9s + 1)(13.4s + 1)e^{-59.5s}}{(404s + 1)(11.2s + 1)} - \frac{3.86(11.2s + 1)(80.1s + 1)e^{-5.35s}}{(474s + 1)(14.1s + 1)}$$

$$I_{32} = \frac{0.122(66.9s + 1)(13.4s + 1)e^{-65.7s}}{(23.2s + 1)(404s + 1)} - \frac{1.31(11.2s + 1)(80.1s + 1)e^{-3.82s}}{(66.9s + 1)(474s + 1)}$$

$$I_{13} = \frac{3.54(66.9s + 1)(5.33s + 1)e^{-1.49s}}{(14.1s + 1)(6.41s + 1)} + \frac{22.5(6.41s + 1)(56s + 1)e^{-58.9s}}{(2.61s + 1)(404s + 1)}$$

$$I_{23} = \frac{8.49(6.41s + 1)(56s + 1)(0.483s + 1)}{(66.9s + 1)(2.61s + 1)(0.2 \times 0.483s + 1)} + \frac{0.22(66.9s + 1)(5.33s + 1)e^{-1.29s}}{(86.4s + 1)(14.1s + 1)}$$

The decouplers for partial decoupling are designed as follows:

$$I_{12} = \frac{3.02(66.9s + 1)(13.4s + 1)e^{-59.5s}}{(404s + 1)(11.2s + 1)} - \frac{3.86(11.2s + 1)(80.1s + 1)e^{-5.35s}}{(474s + 1)(14.1s + 1)}$$

$$I_{32} = \frac{0.122(66.9s + 1)(13.4s + 1)e^{-65.7s}}{(23.2s + 1)(404s + 1)} - \frac{1.31(11.2s + 1)(80.1s + 1)e^{-3.82s}}{(66.9s + 1)(474s + 1)}$$

$$I_{13} = \frac{2.96(66.9s + 1)e^{-1.53s}}{14.1s + 1} \qquad I_{23} = I_{21} = I_{31} = 0$$

The frequency for fitting is selected as the smallest relay frequency of the three prior relay tests (i.e., 0.847). With the incorporation of the desired decouplers, the three single-loop PI controllers are tuned in the sequence of 1−2−3.

Based on the ultimate frequencies, the multiloop PI/PID system is decomposed into a faster subsystem consisting of loop 1 and a slower subsystem consisting of loops 2 and 3. The tuning sequence is 1−2−3−2−3, as depicted in Figure 8.

For example 2, the control system is paired on the diagonal elements of the transfer function matrix as reported by the original author. Nevertheless, the proposed algorithms are not limited to a particular pairing configuration insofar as the multiloop system is integral stabilizable (Grosdidier et al., 1985).

6. Results and Discussion

The proposed sequential design method employs prior relay tests to determine the tuning sequence such that the number of iterative steps can be kept to a minimum. For the two examples, the performance of the final controller designs are evaluated via the IAE values of all loops for a unit step change in each setpoint. The PI/PID controller parameters in each tuning step are determined from eqs 11 and 12. The robust stability of the closed-loop system is measured by the minimum singular value of $I + (GG_C)^{-1}$ for all frequencies in the face of unstructured uncertainties.

The PI/PID settings of multivariable decoupling controllers and multiloop controllers, the IAE values of each loop, and the indices for robust stability resulting from the sequential designs and other methods are shown in Tables 1−3. For example 1, the proposed method is compared with the Niederlinski method, and for example 2, it is compared with the empirical method (settings reported by the original authors using trial-

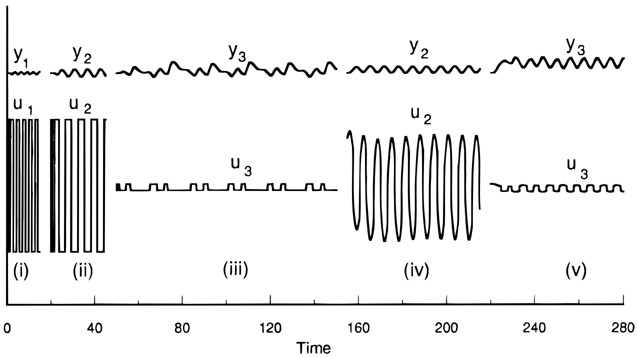


Figure 8. Tuning procedures for multiloop PI design of example 2.

and-error procedures) and the BLT method. On the basis of performance, robust stability, and integrity, the sequential design method is far superior to the other methods, as will be demonstrated later.

The proposed method is applied to design multivariable PID decoupling controllers and multiloop PID controllers for both pairings of example 1. The PID controllers are in the form of series connection with $\alpha = 0.05$. It is compared with the multiloop PID controllers of Niederlinski (1971) in Tables 1 and 2. Figures 9 and 10 compare the setpoint responses produced by the multivariable PID decoupling designs and Niederlinski's designs. It appears that the multivariable decoupling designs are significantly better in performance and robustness for both pairings. Note that the interaction in one loop due to a step setpoint change in the other loop is indeed diminished by the decouplers. The present multiloop PID designs are slightly worse in performance but better in robustness than the multiloop PID designs of Niederlinski. The lack of robustness for Niederlinski's designs is evident in Figure 11, where the setpoint responses produced by Niederlinski's designs become quite oscillatory when α is changed from 0.05 to 0.2, although the present multiloop PID designs still work well.

For the Tyreus column of example 2, the proposed multivariable PI decoupling designs and multiloop PI/PID designs are compared with the multiloop PI designs from the empirical and BLT methods in Table 3.

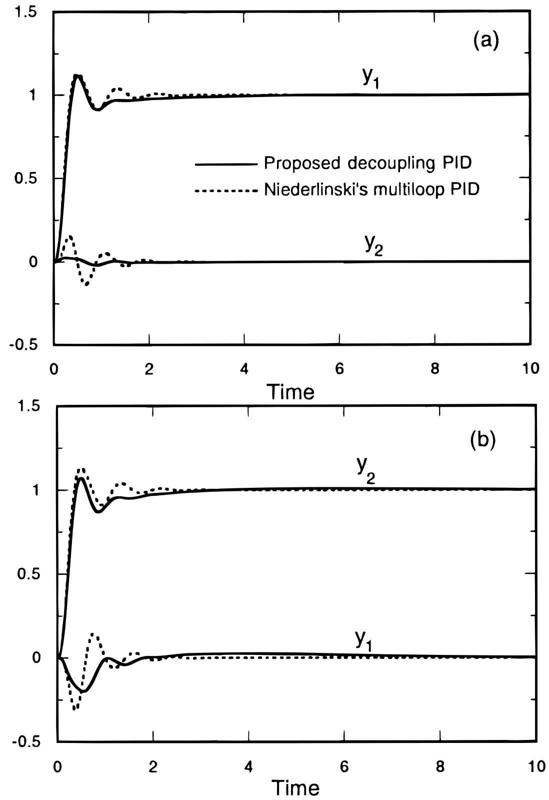


Figure 9. Responses of each loop for example 1 with the first pairing designed using various methods: (a) a step setpoint change in loop 1; (b) a step setpoint change in loop 2.

Apparently, the sequential design method is far superior to the other two in view of performance and robustness. The resulting multivariable PI controller with partial decoupling is undoubtedly the best design. Responses of the closed-loop systems designed by various methods subject to a step setpoint change in loop 2 are shown in Figure 12. Note that the empirical and BLT methods produce rather oscillatory responses, implying poor robustness (the indices for robust stability are merely 0.0681 and 0.0653). Note also that the closed-loop responses of the Tyreus column exhibit multifrequency property, which makes identification of the system

Table 1. Performance and Robustness Results of Various Methods for Example 1 with the First Pairing

	multivariable decoupling PID		proposed multiloop PID		Niederlinski's multiloop PID	
	loop 1	loop 2	loop 1	loop 2	loop 1	loop 2
PID settings						
k_C	3.22	3.24	6.32	7.05	3.20	3.20
τ_I	1.49	1.47	2.21	2.52	0.182	0.182
τ_D	0.0854	0.0845	0.0643	0.0625	0.182	0.182
IAE for unit step change in r_1	0.340	0.0239	0.319	0.142	0.288	0.0965
IAE for unit step change in r_2	0.252	0.378	0.408	0.315	0.154	0.288
minimum singular value	0.707		0.400		0.318	

Table 2. Performance and Robustness Results of Various Methods for Example 1 with the Second Pairing

	multivariable decoupling PID		proposed multiloop PID		Niederlinski's multiloop PID	
	loop 1	loop 2	loop 1	loop 2	loop 1	loop 2
PID settings						
k_C	5.84	3.35	2.91	0.345	1.50	0.320
τ_I	0.503	1.58	0.922	0.670	0.314	0.314
τ_D	0.0425	0.0409	0.139	0.170	0.314	0.314
IAE for unit step change in r_1	0.164	0.0458	0.553	0.932	0.597	0.698
IAE for unit step change in r_2	0.0156	0.188	0.144	0.621	0.149	0.641
minimum singular value	0.595		0.242		0.162	

Table 3. Performance and Robustness Results for Example 2

	proposed PI with complete decoupling			proposed PI with partial decoupling			proposed multiloop PI			proposed multiloop PID			empirical			BLT		
	loop 1	loop 2	loop 3	loop 1	loop 2	loop 3	loop 1	loop 2	loop 3	loop 1	loop 2	loop 3	loop 1	loop 2	loop 3	loop 1	loop 2	loop 3
PID settings																		
k_C	13.7	5.00	0.178	23.6	4.95	0.459	23.6	7.29	0.272	28.3	8.76	0.418	20.0	7.00	0.400	11.3	3.52	0.182
τ_I	44.2	57.3	93.7	66.8	57.2	11.4	66.8	46.1	97.1	66.9	48.7	99.5	5.00	7.00	7.00	7.09	14.5	15.1
τ_D										0.355	0.705	0.824						
IAE for unit step change in r_1	4.72	1.18	7.23	1.79	4.29	5.15	2.60	1.66	5.92	2.40	1.37	3.60	10.0	11.9	20.7	9.91	9.36	34.3
IAE for unit step change in r_2	1.42	17.7	67.4	0.511	8.46	14.6	4.93	6.10	131	3.82	4.85	86.9	12.6	22.3	38.3	24.7	39.2	143
IAE for unit step change in r_3	0.312	0.924	9.77	0.447	7.29	8.57	1.89	2.43	9.58	1.60	2.12	6.81	9.92	16.1	27.5	8.67	12.2	46.4
minimum singular value		0.436			0.293			0.322			0.450			0.0681			0.0653	

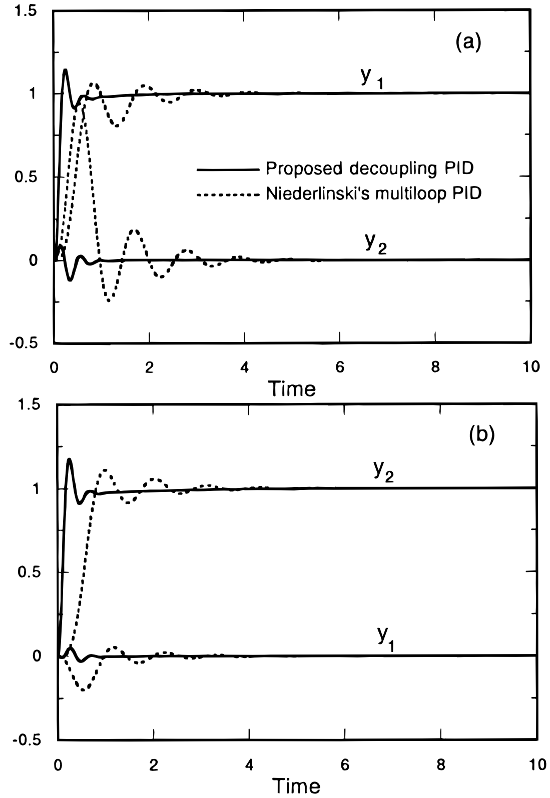


Figure 10. Responses of each loop for example 1 with the second pairing designed using various methods: (a) a step setpoint change in loop 1; (b) a step setpoint change in loop 2.

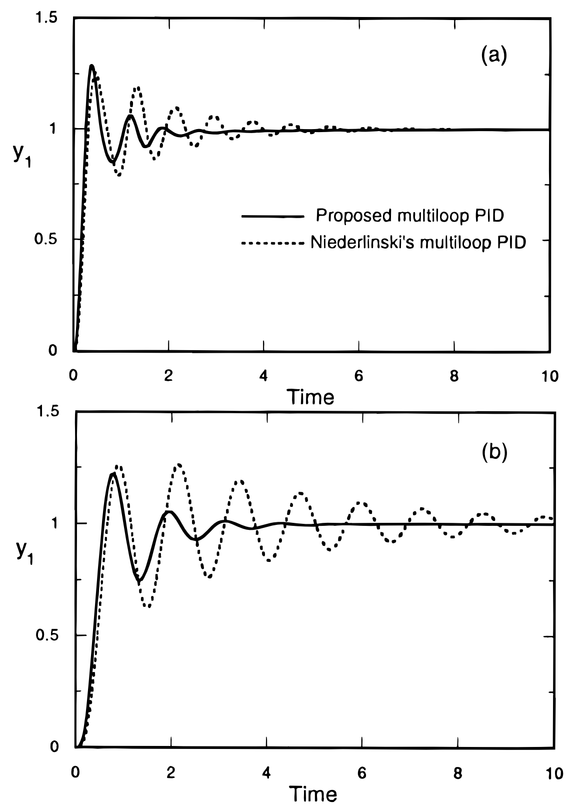


Figure 11. Comparison of various methods for robustness with respect to a change in the parameter α from 0.05 to 0.2: (a) first pairing; (b) second pairing.

extremely difficult. The proposed relay schemes, however, can still easily extract useful information for controller tuning.

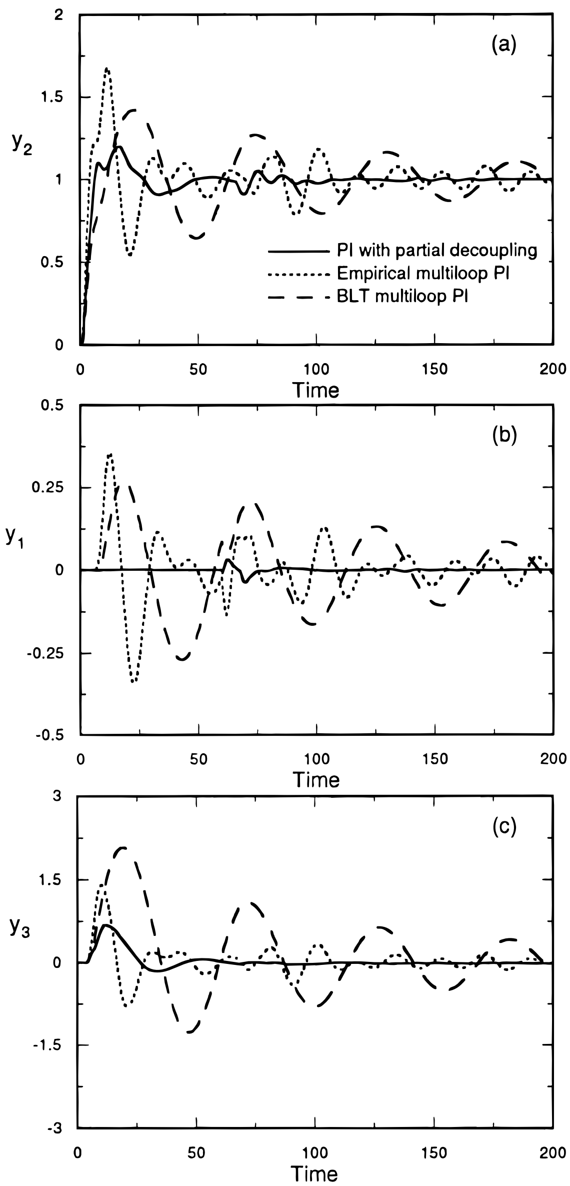


Figure 12. Responses of each loop for example 2 (Tyreus column) designed using various methods subject to a step setpoint change in loop 2.

Note that during the tuning phase (*iii*) shown in Figure 8, the output limit cycle y_3, showing two peaks in one cycle, does not resemble a sinusoidal wave at all. Hence, the conventional relay technique would fail for identification. The proposed biased relay schemes, however, still yield a suitable model without difficulty.

The issue of integrity is considered with example 2. A multivariable control system is said to have satisfactory integrity if it remains stable under all combinations of a stipulated set of failure conditions (Postlethwaite et al., 1981). The set of primary concern includes sensor and actuator failures. Clearly, any design techniques aim at devising feedback controllers for practical systems must incorporate a stability check for such component breakdowns. Figure 13 shows that with the failure of loop 1 (the loop is unexpectedly open), the Tyreus column under the empirical design becomes unstable, indicating its lack of integrity. The proposed multiloop PI design, on the contrary, produces even more stable responses.

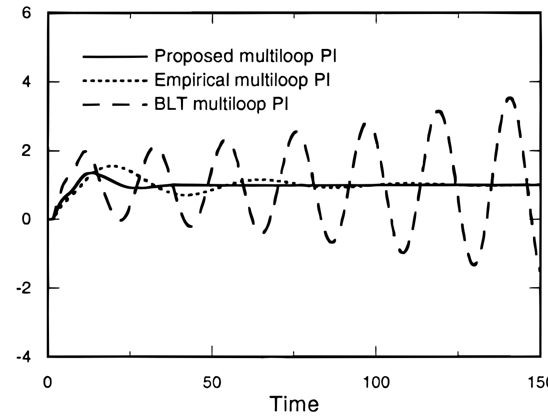


Figure 13. With loop 1 suddenly open, setpoint responses of loop 3 for Tyreus column designed using various methods.

7. Conclusions

A method is developed to design multivariable decoupling and multiloop PI/PID controllers in a sequential manner. The method is based on a single-loop tuning technique developed for multivariable systems with unknown dynamics. The design algorithms lead to good performance, robust stability, and integrity.

The proposed single-loop tuning technique is composed of biased relay schemes and tuning formulae. A first-order-plus-time-delay model with a wide range of d/τ is employed to account for versatile dynamics encountered in a multivariable environment. The biased relay schemes, as modification of the conventional relay method, can quickly yield the appropriate model and are robust with respect to static disturbances during the identification phase. Unlike the conventional relay technique, they require only the existence of sustained oscillations regardless of the shape of excited limit cycles. This feature is very suitable for identification of multivariable systems in a sequential fashion.

Prior biased relay tests on each individual loop with the other loops open are essential to the sequential design. With such knowledge to distinguish the loop speeds by their ultimate frequencies, the most efficient tuning sequence is developed *a priori*, which guarantees the design procedure with minimal engineering effort. It should be emphasized that even though the most efficient tuning sequence is given, the sequential design method is not limited to that sequence. In effect, any tuning sequence can be employed except that more iteration steps might be required.

The proposed algorithms do not address the variable pairing problem. They are in effect not restricted to a particular pairing configuration. They always lead to stable design insofar as the multivariable system with an arbitrary pairing is integral stabilizable.

The underlying theme of this work is to provide plant operators with easy-to-understand methods for quickly achieving satisfactory control over unknown multivariable systems. Despite their simplicity, the proposed methods yield multivariable designs far superior to that resulting from the empirical method based on a time-consuming trial-and-error procedure. The proposed methods can surely be valuable for practical applications.

Acknowledgment

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Nomenclature

B = bias transfer function
D_i = controller transfer function of loop i defined in eq 2b
d = time delay
d_{ij} = time delay of g_{ij}
G_P = process transfer matrix
g = process transfer function
g_C = controller transfer function
g_{ij} = i, j element of G_P
h = magnitude of relay
I_{ij} = transfer function of decoupler from loop j to loop i
K_i = controller transfer function of loop i defined in eq 2a
k_C = proportional controller gain of series connection
k'_C = proportional controller gain of parallel connection
k_{Ci} = proportional controller gain of loop i
k_{ij} = steady-state gain of g_{ij}
k_P = process steady-state gain
L = time of prediction
P_u = ultimate period
r_i = reference variable of loop i
T = period of limit cycles
t_0 = an arbitrary time after the transients die out
u_i = input variable of loop i
v_i = manipulated variable of loop i
y_i = output variable of loop i
α = parameter defined in eq 2b
β = parameter defined in eq 10
γ = parameter defined in eq 10
τ = time constant
τ_C = filter time constant for IMC−PID design
τ_D = derivative time of series connection
τ'_D = derivative time of parallel connection
τ_{Di} = derivative time of series connection for loop i
τ_I = integral time of series connection
τ'_I = integral time of parallel connection
τ_{Ii} = integral time of series connection for loop i
τ_{ij} = time constant of g_{ij}
ω_o = relay frequency

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