

Contents

	Definitions and Symbols	15
0.	Summary of Matrix Algebra and Allied Topics	
0.1	Definitions	17
0.2	Elementary Operations	18
1.	Linear Programming	
1.1	General Methods	
1.1.1	The Primal Simplex-Algorithm	22
1.1.2	The Two-Phase Method	25
1.1.3	The Primal Simplex-Algorithm without Explicit Identity Matrix	30
1.1.4	The Dual Simplex-Algorithm	34
1.1.5	Sensitivity Analysis and Parametric Programming (S.A. and P.P.).....	38
1.1.5.1	<i>S.A. and P.P. with Expected Alterations</i>	39
1.1.5.2	<i>S.A. and P.P. with Unexpected Alterations</i>	
1.1.5.2.1	Subsequent Alterations of the Restriction Vector	43
1.1.5.2.2	Subsequent Alterations of Coefficients of the Objective Function.....	45
1.2	Shortened Methods	
1.2.1	The Transportation Problem.....	48
1.2.1.1	The Northwest-Corner Rule.....	49
1.2.1.2	The Row Minimum Method.....	51
1.2.1.3	The Column Minimum Method.....	54
1.2.1.4	The Matrix Minimum Method.....	57
1.2.1.5	The Double Preference Method.....	60
1.2.1.6	VOGEL's Approximation Method (VAM).....	66
1.2.1.7	The Frequency Method.....	71
1.2.1.8	The Stepping-Stone Method.....	72
1.2.2	The Hungarian Method (Kuhn)	75

1.2.3	The Decomposition Principle (Dantzig; Wolfe).....	81
1.2.4	FLOOD's Technique	89
1.3	Theorems and Rules	
1.3.1	The Dual Problem	91
1.3.2	Theorems of Duality	93
1.3.3	The Lexicographic Selection Rule.....	94
2.	Integer Programming	
2.1	Cutting Plane Methods	
2.1.1	The GOMORY-I-All Integer Method	96
2.1.2	The GOMORY-II-All Integer Method	100
2.1.3	The GOMORY-III-Mixed Integer Method	103
2.1.4	The GOMORY-III-Mixed Integer Method with Intensified Cuts	106
2.1.5	The Primal Cutting Plane Method (Young; Glover; Ben-Israel; Charnes).....	108
2.2	Branch and Bound Methods	
2.2.1	The Method of LAND and DOIG	111
2.2.2	The Method of DAKIN	118
2.2.3	The Method of DRIEBEEK	122
2.2.4	The Additive Algorithm (Balas).....	129
2.3	Primal-Dual Methods	
2.3.1	A Partitioning Procedure for Mixed Integer Problems (Benders).....	134
3.	Theory of Graphs	
3.0.1	Definitions	143
3.0.2	The Determination of Rank in Graphs.....	146
3.0.3	The Number of Paths in a Graph.....	148
3.0.4	The Determination of the Strongly Connected Components of a Graph.....	149
3.1	Shortest Paths in Graphs	
3.1.1	The Algorithm of DIJKSTRA	151
3.1.2	The Algorithm of DANTZIG	154

3.1.3	The FORD Algorithm I (shortest path(s))	159
3.1.4	The FORD Algorithm II (longest path(s))	160
3.1.5	The Tripel Algorithm	162
3.1.6	The HASSE Algorithm	166
3.1.7	The Cascade Algorithm	168
3.1.8	The Algorithm of LITTLE	169
3.1.9	The Method of EASTMAN	174
3.2	Flows in Networks	
3.2.1	The Algorithm of FORD and FULKERSON	178
3.2.2	The Algorithm of BUSACKER and GOWEN	183
3.2.3	The Method of KLEIN	187
3.2.4	The Out-of-Kilter Algorithm (Ford; Fulkerson)	191
3.3	Shortest Spanning Subtrees of a Graph	
3.3.1	The Method of KRUSKAL	200
3.3.2	The Method of SOLLIN	203
3.3.3	The Method of WOOLSEY	205
3.3.4	The Method of BERGE	207
3.4	Gozinto Graphs	210
3.4.1	The Method of VAZSONYI	210
3.4.2	The Method of TISCHER	212
3.4.3	The Method of FLOYD	213
3.4.4	The Gozinto List Method	214
4.	Planning Networks	
4.0.1	The Critical Path Method (CPM)	217
4.0.2	The CPM Project Acceleration	220
4.0.3	The Program Evaluation and Review Technique (PERT)	224
4.0.4	The Metra Potential Method (MPM)	227
4.0.5	The Graphical Evaluation and Review Technique (GERT)	230

5.	Game Theory	
5.1	Non Matrix Games	
5.1.1	The Normal Form	236
5.1.2	NASH's Solution of the Bargaining Problem	241
5.1.3	The Extensive Form	242
5.2	Matrix Games	
5.2.1	A Method for Determining Pure Strategy Pairs for Two-Person Zero-Sum Games	251
5.2.2	A Method for Solving Two-Person Zero-Sum Games with the Simplex-Algorithm	253
5.2.3	An Approximization Method for Two-Person Zero-Sum Games ("learning method"; Gale; Brown).....	256
5.2.4	The LEMKE-HOWSON Algorithm for the Solution of Bimatrix Games	260
5.3	Decisions under Uncertainty (games against nature)	264
5.3.1	The Solution of WALD.....	265
5.3.2	The Solution of HURWICZ	266
5.3.3	The Solution of SAVAGE and NIEHANS	266
5.3.4	The Solution of BAYES	267
5.3.5	The Solution of LAPLACE	268
5.3.6	The Solution of HODGES and LEHMANN	268
6.	Dynamic Programming	
6.0.1	The n-Period Model	270
6.0.2	The Infinite-Period Model (policy iteration routine)	276
7.	Queueing Models	280
7.0.1	The 1-Channel, 1-Stage Model	282
7.0.2	The 1-Channel, r-Phase Model	284
7.0.3	The k-Channel, 1-Stage Model	285
8.	Nonlinear Programming	
8.1	Theorems and Special Methods	
8.1.1	The Theorem of KUHN and TUCKER	288

8.1.2	The Method of LAGRANGE	289
8.1.3	A Method for the Optimization of Nonlinear Separable Objective Functions under Linear Constraints	292
8.2	General Methods	
8.2.1	The Method of WOLFE (short form)	296
8.2.2	The Method of FRANK and WOLFE	302
8.2.3	The Method of BEALE	307
8.2.4	An Algorithm for the Solution of Linear Complementarity Problems (Lemke)	312
8.2.5	The Gradient Projection Method (Rosen)	315
9.	Generation of Random Numbers (simulation)	321
9.0.1	The AWF-Cubes (Graf)	321
9.0.2	The Midsquare Method (J.v.Neumann)	322
9.0.3	A Mixed Congruence Method	323
9.0.4	A Multiplicative Congruence Method	324
10.	Replacement Models	
10.1	Replacement Models with Respect to Gradually Increasing Maintenance Costs	
10.1.1	A Model Disregarding the Rate of Interest	325
10.1.2	A Model Regarding the Rate of Interest	326
10.2	Replacement Models with Respect to Sudden Failure	
10.2.1	A Model Disregarding the Rate of Interest	328
10.2.2	A Model Regarding the Rate of Interest	331
11.	Inventory Models	333
11.0.1	The Classical Inventory Model (Andler)	333
11.0.2	An Inventory Model with Penalties for Undersupplied Demands	334
11.0.3	An Inventory Model with Terms for Delivery	335
11.0.4	An Inventory Model with Damage to Stock	337
11.0.5	An Inventory Model with Rebates (different price intervals)	338

11.0.6	An Inventory Model with Respect to Transportation Capacity	340
12.	Sequencing Models	
12.0.1	JOHNSON's Algorithm for Two Machines	343
12.0.2	JOHNSON's Algorithm for Three Machines (special case)	345
12.0.3	A Heuristic Solution for a Sequencing Problem	347
13.	Plant Location Models	
13.1	Exact Methods	
13.1.1	The Optimal Plant Location in a Transportation Network I	350
13.1.2	The Optimal Plant Location in a Transportation Network II	351
13.1.3	The Optimal Plant Location on a Straight Line	353
13.1.4	The Optimal Plant Location with Respect to Rectangular Transportation Movements	354
13.2	Heuristic Methods	
13.2.1	The Center of Gravity-Method	356
13.2.2	A Solution by Vector Summation	358
13.2.3	An Iterative Method	363
	Appendix	367
	Table 1 : $q^k = (1 + i)^k$	367
	Table 2 : $q^{-k} = (1 + i)^{-k}$	367
	Table 3 : e^{-k}	368
	Table 4 : Random numbers with an equal distribution.....	370
	Table 5 : Area under the standardized normal distribution function.....	372
	Bibliography	373