

# Contents

PREFACE	<i>v</i>
<b>1</b> PREPARATION FOR THE STUDY OF TRIGONOMETRY	<b>1</b>
1.1 Introduction	<i>1</i>
1.2 Sets	<i>1</i>
1.3 Sets of Points on a Line. Coordinates	<i>3</i>
1.4 Order Relations	<i>5</i>
1.5 Variable on a Set. Number Intervals	<i>6</i>
1.6 A Rectangular Coordinate System	<i>9</i>
1.7 Other Plane Figures	<i>11</i>
1.8 The Pythagorean Theorem	<i>14</i>
1.9 The Distance Between Two Points	<i>19</i>
<b>2</b> DIRECTED ANGLES AND THEIR MEASURES	<b>24</b>
2.1 Directed Angles	<i>24</i>
2.2 Units of Angle Measure	<i>24</i>
2.3 Change of Unit of Angle Measure	<i>26</i>
2.4 The Central Angle and the Intercepted Arc	<i>29</i>
2.5 Linear and Angular Speed of Rotation	<i>30</i>
2.6 Standard Position of an Angle	<i>32</i>

<b>3</b>	<b>THE TRIGONOMETRIC FUNCTIONS</b>	<b>37</b>
3.1	Functions	37
3.2	The Trigonometric Functions	39
3.3	Graphical Approximations of the Functional Values	43
3.4	Exact Values of the Functions for Special Angles	46
3.5	Values of Trigonometric Functions from Tables	52
3.6	Values of the Functions of Any Angle. Reduction to Functions of Positive Acute Angles	56
3.7	Trigonometric Functions with Domain the Real Numbers	59
<b>4</b>	<b>VARIATION AND GRAPHS OF THE TRIGONOMETRIC FUNCTIONS</b>	<b>64</b>
4.1	The Sine and the Cosine Functions. Periodic Functions	64
4.2	The Graphs of $y = a \sin bx$ and $y = a \cos bx$	70
4.3	The Tangent and the Cotangent Functions	75
4.4	The Secant and the Cosecant Functions	78
4.5	Composite Functions. Addition of Ordinates	79
4.6	The Graph of $y = \frac{\sin x}{x}$	81
4.7	Points in Polar Coordinates	85
4.8	Suggestions for Rapid Sketching in Polar Coordinates	87
<b>5</b>	<b>FUNDAMENTAL TRIGONOMETRIC IDENTITIES AND EQUATIONS. INVERSE RELATIONS</b>	<b>93</b>
5.1	Equations and Identities	93
5.2	Verifying an Identity	94
5.3	Eight Fundamental Trigonometric Identities	97
5.4	Simplifying Trigonometric Expressions. Building More Identities	99
5.5	Solution of Elementary Trigonometric Equations	104
5.6	Inverse Relations	110
5.7	The Inverse Trigonometric Functions	113

<b>6</b>	<b>SOLUTION OF RIGHT TRIANGLES AND APPLICATIONS</b>	<b>120</b>
6.1	Introduction	120
6.2	Solving the Right Triangle	121
6.3	Logarithms of the Trigonometric Functions	125
6.4	Use of Logarithms in Solving Right Triangles	125
6.5	Applications	128
<b>7</b>	<b>FURTHER TRIGONOMETRIC IDENTITIES AND EQUATIONS</b>	<b>135</b>
7.1	Introduction	135
7.2	The Sine and Cosine Addition Formulas. Related Identities	136
7.3	The Tangent and Cotangent Addition Formulas	144
7.4	The Double-Angle and Half-Angle Identities	146
7.5	Sums or Differences of Sines or of Cosines	151
7.6	Trigonometric Equations	154
<b>8</b>	<b>THE GENERAL TRIANGLE</b>	<b>168</b>
8.1	Solution of the General Triangle	168
8.2	The Law of Cosines	168
8.3	The Law of Sines	172
8.4	Other Formulas Used in Triangle Solution	180
8.5	Cyclical Permutations in the Triangle Formulas	185
<b>9</b>	<b>COMPLEX NUMBERS</b>	<b>190</b>
9.1	The Complex-Number System	190
9.2	Graphical Representation of Complex Numbers. Polar Form	193
9.3	The Product of Two or More Complex Numbers in the Polar Form. De Moivre's Formula	198
9.4	The Quotient of Two Complex Numbers in Polar Form	201
9.5	Roots of Complex Numbers	201

APPENDIX	209
Accuracy of Computed Results	209
Common Logarithms	213
TABLES	229
I. Squares, Square Roots, Cubes, and Cube Roots	231
II. Logarithms of Numbers	232
III. Natural Functions	234
IV. Logarithms of Sines, Cosines, Tangents, and Cotangents	240
V. Natural Functions, with Radian Measure	246
ANSWERS TO SELECTED EXERCISES	249
INDEX	259