# CONTENTS

#### PART I Introduction to Scientific Measurement

1.	The Meaning of Measurement	
	A Definition of Measurement	3
	Dimensional Analysis	5
	Testing Hypotheses	6
2.	The Conduct of an Experimental Investigation	7
	Design	7
	Crucial Times During an Investigation	8
3.	The Scientific Report	9
	Functions of the Scientific Report	9
	Components of the Scientific Report	10
4.	Procedure in a Laboratory and the Laboratory Notebook	12
	The Laboratory Notebook	12
5.	Experimental Errors	14
	Random Errors	14
	Systematic Errors	15
	Blunders	16
	Analysis of Errors	16
6.	Rejection of Data: Chauvenet's Criterion and Its Dangers	17
7.	The Philosophy of Sampling and the Definition of Statistical	
	Concepts	19
	Population Parameters	19
	Variation and Distributions	20
•	Functions of Statistics	21
	Definitions of Basic Statistical Concepts	21
8.	Discussion of the Analysis of Samples	30
9.	Discussion of Discrete and Continuous Frequency Distributions and	
	Histograms	32

	Normalized Frequency Distribution	32
	Normalized Frequency Histogram	33
	Definitions	35
	Skewness	37
	Sheppard's Correction for Grouping Data	37
10.	Propagation of Error and Least Squares	-39
	General Case of Error Propagation	39
	Independent Errors	40
	Graphical Description of Error Propagation	41
	Minimum Variance (Least-Squared Error)	42
	Nonindependent or Correlated Errors	43
	Covariances of Calculated Quantities	45
	Generalization of (10.3)	45
	Error Propagation with Complex Variables	47

#### PART II

# Introduction to Graphical Techniques and Curve Fitting

11.	Display Graphs	51
	Horizontal Bar Chart	51
	Pie Chart Or Area Diagram	51
	Volumetric or Solid Diagram	52
12.	Correlation Graphs	53
	Silhouette Chart	53
	Horizontal Bar Chart	54
	Vertical Bar Chart	54
	Line Chart	54
	Search for Correlations	55
	Correlation Coefficient	62
13.	Graphs Relating More than Two Variables	63
	Use of Perspective	63
	Projections	63
	Contour Plots	64
14	Straight-Line Graphs and Fitting	71
	Straight Line	71
	Calculation of the Least-Squares Straight Line	74
	Fitting Straight-Line Data When Both Variables Have Uncertainties	75
15.	Reduction to Straight-Line Graphs	76
	Log Plots	76
	Semilog Plots	79
-	General Use of Logarithmic Scales	87
16.	Calculating Charts	90
,	Fixed Scales	90
	Sliding Scales	90
	Nomograms	01

#### PART III Probability

17.	The Meaning of Probability	101
	Random Phenomena and Random Variables	101
	Probability Distributions and Their Description	102
	Chebychev's Inequality	105
	Derivation of Chebychev's Inequality	106
	Symmetrical and Asymmetrical Distributions	106
	Different Kinds of Probability	107
18.	Some Arithmetic on Combinations and Permutations	111
	Arrangements: Permutations and Variations	111
	Combinations	112
	The Binomial Theorem	112
	The Laplace Triangle	113
	Remarks on the Factorial Function	115
	Some Necessary Specifications in Combinatorial Analysis	116
	The Multinomial Theorem	118
19.	Event Calculus—The Logic of Probability	121
	Definitions	121
	Conditional Probabilities: Dependence and Independence	125
	Expectation Values—Recursion Relations	136
20.	Joint Probability Distributions and Functions of Random Variables	139
	Joint and Marginal Probability Distributions	139
	Expectation Values	140
	Independence	141
	Covariance	142
	Variance	144
	Calculus of Probability Density Functions (Univariate)	147
	Calculus of Probability Density Functions (Multivariate)	148
21.	Geometrical Probability, Random Numbers, and Monte Carlo	
	Experiments	153
	Buffon's Needle	153
	Bertrand's Paradox	156
	Randomness and Random Numbers Drawn from the Uniform	
	Distribution	157
	Simulation of Probability Problems: Monte Carlo Experiments	158
	Operations with Random Numbers	162
	Sums of Random Numbers	162
	Random Numbers Drawn from an Arbitrary Distribution	167
	Correlations	169

#### PART IV

### Some Probability Distributions and Applications

22.	The Binomial Distribution	181
	Definitions	181
	Reproductive Property of the Binomial Distribution	183
	Probability of a Range of Values	183

	Symmetry and Asymmetry	184
	Expectation Value of the Binomial Distribution	185
	Variance of the Binomial Distribution	186
	The Expectation Value for the Number of Trials Required	
	for a Specified Number of Successes	186
	The Mode of the Binomial Distribution	187
	Rare Events :	189
	Inverse Probability	189
	Law of Large Numbers	190
	The Frequency Definition of Probability	191
	The Law of Large Numbers and the Sample Mean	192
23.	The Hypergeometric Distribution	193
	Definition of the Probability	193
	Expectation Value and Variance	1 <b>94</b>
	Binomial Approximation to the Hypergeometric Distribution	199
	Inverse Probability	200
24.	The Poisson Distribution	202
	Exact Model	202
	Poisson Approximation to the Binomial Distribution	207
	Expectation Value for the Poisson Distribution	212
	Variance for the Poisson Distribution	212
	Poisson Approximation to the Hypergeometric Distribution	212
	Reproductive Property of the Poisson Distribution	213
	Radioactive Decay and the Exponential Decay Distribution	213
	Interval Distribution	214
	Interval Distribution	216
	Cumulative Poisson Distribution Function	210
25	The Gaussian or Normal Distribution	219
20.	Derivation of the Gaussian Distribution from Certain	223
	Assumptions	223
	Relation of the Mean Deviation to the Standard Deviation	223
	Derivation of Gaussian Distribution from the Binomial Distribution	220
	Derivation of Gaussian Distribution from the Poisson Distribution	220
	Inverse Probability	228
	Some Properties of the Normal Distribution	229
	Normal Deviate Test for the Difference of Two Sample Means	235
	Normal Approximation to the Binomial Distribution	237
	The Central Limit Theorem (Normal Convergence Theorem)	244
26.	The Chi-Square Distribution	254
	Chi-Square and Minimization	254
	Probability Density Functions for <i>n</i> Independent Degrees	
	of Freedom	259
	Mean, Mode, and Variance of the Chi-Square Distribution	261
	Computations with the Chi-Square Distribution	263
	Approximation to the Chi-Square Distribution	264
	The Sample Variance	269
27.	Student's t Distribution	274
	Definition of $t$ and its p.d.f.	274
	Cauchy Distribution	275
	Applications of Student's t Distribution	279

28.	Miscellaneous Other Probability Distrib	outions and Examples	283
	The Negative Binomial Distribution	-	283
	The Multinomial Distribution		284
	The Exponential Distribution		284
	The Weibull Distribution		284
	The Log-Normal Distribution		285
	The F-distribution	n de la companya de La companya de la comp	285
	Folded Distributions		286
	Folded Normal Distribution		287
	Truncated Distributions		287
	Truncated Normal Distribution	•	287
	The Bivariate Normal Distribution		288
	Multivariate Normal Distribution		290

### PART V Statistical Inference

29.	Estimation	293
	Confidence Intervals	294
	Estimation of a Population Mean for a Large, Homogeneous	
	Population	295
	Estimation of Population Mean for a Finite Population	296
	Stratified Sampling: Estimation of a Population Mean where the	
	Population is Large and Partitioned into Strata	297
	Estimation of a Probability (Binomial)	307
	Estimation of a Population Proportion	309
30.	Estimation and the Method of Maximum Likelihood	311
	Likelihood Estimators	311
	General Properties of Estimators	317
	Nonanalytical Solution of the Likelihood Equation	326
	Asymptotic Properties of the Likelihood Function	329
	Finite Data	330
	Error Expected Prior to a Measurement	331
	Inefficient Statistics	332
	Graphical Methods: the Score Function	333
31.	Hypothesis Testing and Significance	340
	Kinds of Hypotheses	340
	Consistency and Proof	340
	Two Kinds of Error and the Cost of Being Wrong	341
	Concepts in Hypothesis Testing	342
	The Neyman-Pearson Theorem	350
	The Likelihood Ratio	352
	The Generalized Likelihood Ratio	352
	Large-Sample Properties of the Likelihood Ratio	354
	The Generalized $\chi^2$ Test for Goodness-of-Fit	356
	Use of the $\chi^2$ Test for Goodness-of-Fit	. 352
32.	Chi-Square Minimization Methods	351
	Review of $\chi^2$	359
	Une-Parameter $\chi^{-1}$ : c = c <sub>1</sub>	361
	Multiparameter $\chi^{-}$ : $\mathbf{c} = (c_1, \ldots, c_r)$	364

Least-Squares Methods; Curve Fitting	387
General Formulation	387
Linear Case	388
Goodness-of-Fit	397
Linear Least Squares with Linear Constraints	397
Nonlinear Least Squares	399
	Least-Squares Methods; Curve Fitting General Formulation Linear Case Goodness-of-Fit Linear Least Squares with Linear Constraints Nonlinear Least Squares

#### APPENDICES

А.	Review of Notation and Some Elementary Mathematics	401
	Maxima and Minima of Functions	405
<b>B</b> .	Matrices, Determinants, and Linear Equations	407
I.	Units and Standards of Weights and Measures	419
II.	Dimensional Analysis	431
III.	Some Comments on the Factorial, Gamma, and Error Functions	434
	Stirling's Formula	436
	Gamma and Beta Functions	437
IV.	Eight Hundred Uniformly Distributed Random Numbers and Eight	
	Hundred Random Normal Deviates	439
V.	Tables of the Negative Exponential $e^{-x}$	449
VI.	Tables of the Gaussian (Normal) Distribution	457
VII.	Tables and Graphs of the Chi-Square Distribution	465
VIII.	Tables of the Student's t Distribution	483
Guide	for Further Reading and Bibliography	449
Index		505