519 KEN 2nd ed. Pt.2

CONTENTS

CHAPTER PAGE

I PROBABILITY 1

1.1 Importance. — 1.2 The Classical Definition of Probability. — 1.3 The Frequency Definition. — 1.4 Other Definitions of Probability. — 1.5 Some Theorems of Algebra. — 1.6 Some Fundamental Theorems of Probability. — 1.7 The Axiomatic Treatment of Probability. — 1.8 Some Examples of Probability. — 1.9 Continuous Probabilities. — 1.10 Bayes' Theorem. — 1.11 Bayes' Theorem for Future Events. — Problems. — References.

II THE BINOMIAL DISTRIBUTION AND THE NORMAL AND POISSON APPROXIMATIONS 22

2.1 The Binomial or Bernoulli Distribution. — 2.2 Graphical Representation. — 2.3 Frequency Functions. Stieltjes Integrals. — 2.4 Distribution Functions. — 2.5 Mathematical Expectation. — 2.6 Moments. — 2.7 The Cauchy Distribution. — 2.8 Moments of the Bernoulli Distribution. — Problems. — 2.9 Approximating the Binomial with the Normal Curve. — 2.10 The DeMoivre-Laplace Theorem. — 2.11 Simple Sampling of Attributes. — 2.12 Bernoulli's Theorem. — 2.13 Tables of the Normal Law. — 2.14 The Poisson Exponential Approximation. — 2.15 Poisson and Lexis Schemes. — 2.16 The Hypergeometric Distribution. — Problems. — References.

III SOME USEFUL INTEGRALS AND FUNCTIONS 55

3.1 Some Properties of Definite Integrals. — 3.2 The Gamma Function. — 3.3 Stirling's Approximation. — 3.4 The Beta Function. — 3.5 Reduction to Gamma and Beta Functions. — 3.6 Incomplete Beta and Gamma Functions. — Problems. — References.

IV DISTRIBUTIONS OF TWO OR MORE VARIABLES; MOMENT-GENERATING FUNCTIONS; THE LAW OF LARGE NUMBERS 67

4.1 Joint Distribution of Two Variables. — 4.2 Moments. — 4.3 Discrete Variables. — 4.4 Joint Distribution of More than Two Variables. — 4.5 Some Theorems on Expectation. — 4.6 Moment-generating and Characteristic Functions. — 4.7 Some Examples of Moment-generating Functions. — 4.8 Change of Scale and Origin. — 4.9 Uniqueness Theorem for Characteristic Functions. — 4.10 Cumulants and the Cumulant-generating Function. — 4.11 Additive Property of Cumulants. — 4.12 Sheppard's Corrections. — 4.13 Orthogonal Linear Transformations. — 4.14 The Bienaymé-Tchebycheff Inequality. — 4.15 The Weak Law of Large Numbers. — 4.16 The Strong Law of Large Numbers. — 4.17 The Central Limit Theorem. — Problems. — References.

CHAPTER PAGE

V THE GAMMA, BETA, AND CHI-SQUARE DISTRIBUTIONS; THE PEARSON AND GRAM-CHARLIER SYSTEMS OF CURVES; CURVE FITTING 94

5.1 The Gamma Distribution. — 5.2 The Beta Distributions. — 5.3 The Chi-square Distribution. — 5.4 Distributions of Sums of Squares. — Problems. — 5.5 Pearson System. — 5.6 Some Pearson Types. — 5.7 Gram-Charlier and Edgeworth Series. — 5.8 Curve Fitting. — 5.9 An Example of Curve Fitting. — 5.10 Approximate Tests of Normality. — 5.11 The Multinomial Distribution. — 5.12 Chi-square as a Measure of Sample Deviation. — 5.13 The Chi-square Test of Hypotheses. — 5.14 The Chi-square Test Applied to Curve Fitting. — 5.15 The Log-normal Distribution. — Problems. — References.

VI FUNDAMENTALS OF SAMPLING THEORY WITH SPECIAL REFERENCE TO THE MEAN 127

6.1 Introduction. — 6.2 Method of Attack. — 6.3 Point Estimation and Interval Estimation. — 6.4 Confidence Belts and Limits. — 6.5 Standard Error of the Mean. — 6.6 Confidence Limits for the Mean. — 6.7 Null Hypothesis and Significance Tests. — 6.8 The Distribution of the Mean. — 6.9 An Experiment. — Directions. — 6.10 Standard Errors of Moments. — 6.11 Sampling from a Finite Parent Population. — 6.12 Size of Sample to Have a Given Reliability. — 6.13 Standard Error of an Observed Proportion. — 6.14 Standard Error of a Difference of Proportions. — 6.15 Confidence Limits for the Parameter of a Binomial Distribution. — 6.16 Sampling from a Finite Binomial Population. — 6.17 Confidence Limits for the Difference of Parameters in Two Binomial Distributions. — 6.18 Confidence Limits for the Poisson Distribution. — 6.19 Picking a Random Sample. — 6.20 Tests for Randomness. — 6.21 Stratified Sampling. — 6.22 Systematic Sampling. — Problems. — References.

VII SMALL OR EXACT SAMPLING THEORY 160

7.1 Introduction. — 7.2 Expected Value of s^2. — 7.3 Degrees of Freedom. — 7.4 Standard Error of the Variance. — 7.5 The Distribution of s^2 in Samples from a Normal Population. — 7.6 The Analytical Approach. — 7.7 The Geometrical Derivation of the Sampling Distribution of Variance. — 7.8 The Distribution of the Standard Deviation. — 7.9 The "Best" Estimate of the Population Standard Deviation. — 7.10 The $(\bar{x}, s)$-Frequency Surface. — 7.11 "Student's" Distribution. — 7.12 Student's t-Distribution. — 7.13 Difference Between Two Means. — 7.14 Fisher's z-Distribution. — 7.15 Significance of Difference Between Variances. — 7.16 The Distribution of F. — 7.17 Confidence Limits. — 7.18 Standard Errors of the k-Statistics and of g_1 and g_2. — 7.19 The Distribution of Extreme Values and of the Range. — 7.20 Confidence Limits for the Binomial and Poisson Distributions. — Problems. — References.

VIII LINEAR REGRESSION, SIMPLE CORRELATION, AND CONTINGENCY 199

8.1 Linear Regression. — 8.2 The Standard Error of Estimate. — 8.3 The Normal Correlation Surface. — 8.4 Limiting Forms. — 8.5 Tetrachoric

CHAPTER PAGE

Correlation. — 8.6 Linear Regression as Estimated from a Sample. — 8.7 The Sample Standard Error of Estimate. — 8.8 Confidence Limits for the Constants of the Regression Line. — 8.9 Confidence Limits for the True Regression and for an Estimated y Corresponding to Any Given x. — 8.10 Confidence Limits for x, Given y, When the Regression Is Calculated for Fixed x. — 8.11 Linear Regression with the x's and y's Both Subject to Error. — 8.12 The Distribution of r When $\rho = 0$. — 8.13 Confidence Limits for the Variance Ratio of Two Correlated Variates. — 8.14 The Distribution of r When ρ Is Not Zero. — 8.15 Fisher's z'-Transformation. — 8.16 Rank Correlation. — 8.17 Kendall's Method of Rank Correlation. — 8.18 Ties in Rank Correlation. — 8.19 Contingency Tables. — 8.20 The $2 \times n$ and 2×2 Contingency Tables. — 8.21 The Exact Distributions for 2×2 Tables. — 8.22 The Combination of Probabilities from 2×2 Tables. — Problems. — References.

IX ANALYSIS OF VARIANCE AND COVARIANCE 238

9.1 Analysis of Variance. One-way Classification. — 9.2 Two-way Classification, with One Individual in Each Sub-class. — 9.3 Interpretation of the Mean Square. — 9.4 Three-way Classification (One Member in Each Sub-class). — 9.5 Assumptions Made in Analysis of Variance. — 9.6 Transformations to Stabilize Variance. — 9.7 Tests of Homogeneity of Variance. — 9.8 The Behrens-Fisher Test. — 9.9 Estimation of Components of Variance. — 9.10 Confidence Limits for the Component of Variance Due to Treatment Effects. — 9.11 Effect of Unequal Numbers in the Sub-classes on the Estimation of Treatment Effects. — 9.12 Interaction with Unequal Sub-class Numbers. — 9.13 Proportional Sub-class Numbers. — 9.14 The Missing Plot Technique. — 9.15 Analysis of Covariance. — 9.16 Experimental Design. — 9.17 Split Plots. Confounding. — 9.18 Intra-class Correlation. — Problems. — References.

X MATRIX ALGEBRA AND THE METHOD OF LEAST SQUARES 289

10.1 Introduction. — 10.2 Normal Equations. — 10.3 Matrix Algebra. — 10.4 Transposition. — 10.5 Matrices and Linear Transformations. — 10.6 The Determinant of a Matrix. — 10.7 The Inverse of a Matrix. — 10.8 Numerical Solution of Normal Equations. — 10.9 Calculation of the Inverse of a Matrix. — 10.10 Moving Decimal Points in Matrix Elements. — 10.11 Non-symmetric Matrix Equations. — 10.12 Improvement of an Approximation to the Inverse Matrix. — 10.13 Variance and Covariance of the Regression Coefficients in Linear Regression. — 10.14 Residuals. — 10.15 Distribution of Sum of Squares of Residuals. — 10.16 Confidence Limits for the True Regression Coefficients. — 10.17 Omission of Variates in Multiple Regression. — 10.18 Solution of a Set of Linear Equations with More Equations than Unknowns. — 10.19 Weighted Observations. — 10.20 Condition Equations or Equations of Constraint. — Problems. — References.

XI CURVILINEAR REGRESSION; MULTIPLE AND PARTIAL CORRELATION 323

11.1 The Correlation Ratio. — 11.2 A Test for Linearity of Regression. — 11.3 Fitting a Polynomial of Second or Higher Order. — 11.4 Orthog-

CHAPTER PAGE

onal Polynomials. — 11.5 Seidel's Method of Successive Approximations. — 11.6 Exponential and Modified Exponential Regression. — 11.7 Fitting a Simple Harmonic Curve to a Series of Observations. — 11.8 Estimation of x for a Given y in Curvilinear Regression. — 11.9 Estimation of Maximum or Minimum in Curvilinear Regression. — 11.10 The Geometrical Picture of Multiple Regression and Correlation. — 11.11 Variance about the Regression Plane. — 11.12 Variance Due to Regression. — 11.13 The Multiple Correlation Coefficient. — 11.14 Some Limiting Cases of Multiple Correlation. — 11.15 The Distribution of the Multiple Correlation Coefficient for Samples from an Uncorrelated Parent Population. — 11.16 The Distribution of the Multiple Correlation Coefficient for a Correlated Parent Population. — 11.17 Partial Correlation. — 11.18 Partial Correlations with k Variables. — 11.19 The Distribution of the Partial Correlation Coefficient. — 11.20 Correlograms. — 11.21 Discriminant Functions. — 11.22 An Alternative Approach to Discriminant Functions. — Problems. — References.

XII FURTHER CONSIDERATIONS ON STATISTICAL INFERENCE 367

12.1 Introduction. — 12.2 The Best Unbiased Estimate of a Parameter. Fisher's Inequality. — 12.3 Consistent and Efficient Statistics. — 12.4 Sufficient Statistics. The Method of Maximum Likelihood. — 12.5 Curve-fitting by the Method of Maximum Likelihood. — 12.6 The Chi-square Test of Goodness of Fit. — 12.7 The Correction of an Inefficient Estimate. — 12.8 The Neyman-Pearson Theory of Confidence Intervals. — 12.9 A Geometrical Illustration of Confidence Intervals. — 12.10 The Determination of Confidence Intervals. — 12.11 Confidence Intervals and Fiducial Inference. — 12.12 Tests of Hypotheses. — 12.13 The Power of a Test. — 12.14 Uniformly Most Powerful Tests. — 12.15 Confidence Regions with More than One Parameter. — 12.16 Composite Hypotheses. — 12.17 The Power of the t-Test. — 12.18 The Power Function of Analysis of Variance Tests. — 12.19 Sequential Tests of Hypotheses. — 12.20 Expected Number of Observations in a Sequential Probability-Ratio Test. — 12.21 Test of a Hypothesis Against a One-sided Alternative. — 12.22 The Sequential Test for a Binomial Distribution. — 12.23 Tolerance Limits for a Parent Population. — Problems. References.

APPENDIX TABLES 407

INDEX 419