## **CONTENTS**

| Preface |        |   | page v     |  |  |  |
|---------|--------|---|------------|--|--|--|
| 1       | Bosons |   |            |  |  |  |
|         | 1.1    | The simple harmonic oscillator                        | 1          |  |  |  |
|         | 1.2    | Annihilation and creation operators                   | 3          |  |  |  |
|         | 1.3    | Coupled oscillators: the linear chain                 | 5          |  |  |  |
|         | 1.4    | Three-dimensional lattices and vector fields          | 9          |  |  |  |
|         | 1.5    | The continuum limit                                   | 12         |  |  |  |
|         | 1.6    | Classical field theory                                | 14         |  |  |  |
|         | 1.7    | Second quantization                                   | 18         |  |  |  |
|         | 1.8    | Klein-Gordon equation                                 | 21         |  |  |  |
|         | 1.9    | Sources of a field, and interactions between fields   | 22         |  |  |  |
|         | 1.10   | Example: Rayleigh scattering of phonons               | 24         |  |  |  |
|         | 1,11   | Example: Yukawa force                                 | 26         |  |  |  |
|         | 1.12   | Charged bosons  | <b>2</b> 8 |  |  |  |
| 2       | Feri   | mions   |            |  |  |  |
|         | 2.1    | Occupation-number representation                      | 32         |  |  |  |
|         | 2.2    | Annihilation and creation operators: anticommutation  | n 33       |  |  |  |
|         | 2.3    | Second quantization                                   | 36         |  |  |  |
|         | 2.4    | Scattering: connection with statistical mechanics     | 39         |  |  |  |
|         | 2.5    | Interactions between particles: momentum conservation | -<br>41    |  |  |  |
|         | 2.6    | Fermion-boson interaction                             | 43         |  |  |  |
|         | 2.7    | Holes and antiparticles                               | 48         |  |  |  |

| ~ | n | N   | т | E | N  | т | c |
|---|---|-----|---|---|----|---|---|
| L | w | 1.4 |   | £ | TA |   | - |

| 2 | Da-  | 4   | hai |     | 4h 0 0 mm |
|---|------|-----|-----|-----|-----------|
| J | Let. | uur | Da  | uon | theory    |

X

4

| 3.1  | The Brillouin–Wigner series                               | page~53 |
|------|---|---------|
| 3.2  | The Heisenberg representation                             | 56      |
| 3.3  | Interaction representation                                | 60      |
| 3.4  | Time-integral expansion series                            | 62      |
| 3.5  | S-matrix  | 64      |
| 3.6  | S-matrix expansion: algebraic theory                      | 67      |
| 3.7  | Diagrammatic representation                               | 74      |
| 3.8  | Momentum representation                                   | 80      |
| 3.9  | The physical vacuum                                       | 86      |
| 3.10 | Dyson's equation and renormalization                      | 90      |
| Gr   | een functions   |         |
| 4.1  | The density matrix  | 94      |
| 4.2  | Equation of motion of density operator                    | 98      |
| 4.3  | Ensembles in thermal equilibrium                          | 99      |
| 4.4  | The Kubo formula  | 101     |
| 4.5  | The one-particle Green function                           | 104     |
| 4.6  | Energy-momentum representation                            | 107     |
| 4.7  | Evaluation of Green functions                             | 110     |
| 4.8  | Two-particle Green functions                              | 112     |
| 4.9  | The hierarchy of Green functions                          | 116     |
| 4.10 | Time-independent Green functions                          | 117     |
| 4.1  | Matrix representation of the Green function               | 120     |
| 4.12 | 2 Space representation of time-independent Green function | 122     |
| 4.13 | 3 The Born series   | 124     |
| 4.14 | The T-matrix  | 127     |
| 4 15 | 5 Evample: impurity states in a metal                     | 131     |

| 5 | Son  | ne aspects of the many-body problem                 |     |
|---|------|---|-----|
|   | 5.1  | Quantum properties of macroscopic systems page      | 135 |
|   | 5.2  | Statistical methods: the Thomas–Fermi approximation | 136 |
|   | 5.3  | Hartree self-consistent field                       | 138 |
|   | 5.4  | The Hartree–Fock method                             | 140 |
|   | 5.5  | Diagrammatic interpretation of Hartree–Fock theory  | 143 |
|   | 5.6  | The Brueckner method                                | 146 |
|   | 5.7  | The dielectric response function                    | 148 |
|   | 5.8  | Spectral representation of dielectric function      | 150 |
|   | 5.9  | Diagrammatic interpretation of dielectric screening | 154 |
|   | 5.10 | The random phase approximation                      | 158 |
|   | 5.11 | The Landau theory of Fermi liquids                  | 162 |
|   | 5.12 | The dilute Bose gas                                 | 167 |
|   | 5.13 | The superconducting state                           | 170 |
| 6 | Rela | ativistic formulations                              |     |
|   | 6.1  | Lorentz invariance                                  | 175 |
|   | 6.2  | Relativistic electromagnetic theory                 | 177 |
|   | 6.3  | The wave equation and gauge invariance              | 180 |
|   | 6.4  | Quantization of relativistic fields                 | 183 |
|   | 6.5  | Spinors   | 187 |
|   | 6.6  | The Dirac equation                                  | 191 |
|   | 6.7  | The Dirac matrices                                  | 193 |
|   | 6.8  | Quantization of the Dirac field                     | 196 |
|   | 6.9  | Interactions between relativistic fields            | 199 |
|   | 6.10 | Relativistic kinematics                             | 203 |
|   | 6.11 | The analytic S-matrix                               | 207 |

Index

## CONTENTS

| • | 1 He | algebra of symmetry                               |      |     |
|---|------|---|------|-----|
|   | 7.1  | Symmetry operations                               | page | 213 |
|   | 7.2  | Representations                                   |      | 215 |
|   | 7.3  | Regular representations of finite groups          |      | 219 |
|   | 7.4  | The orthogonality theorem                         |      | 222 |
|   | 7.5  | Character and class                               |      | 225 |
|   | 7.6  | Product groups and representations                |      | 230 |
|   | 7.7  | Translation groups                                |      | 235 |
|   | 7.8  | Continuous groups                                 |      | 237 |
|   | 7.9  | The rotation group                                |      | 241 |
|   | 7.10 | Irreducible representations of the rotation group |      | 244 |
|   | 7.11 | Spinor representations                            |      | 247 |
|   | 7.12 | SU(2)   |      | 249 |
|   | 7.13 | SU(3)   |      | 254 |
|   |      |   |      |     |

259