

# Table of Contents

Preface

<b>Chapter 1</b>	<b>Theory of an Anisotropic Elastic Continuum</b>	
	Introduction . . . . .	
1.2	Stress and Strain in an Anisotropic Continuum . . . . .	1
1.3	Equations of Motion and Compatibility . . . . .	4
1.4	Generalized Hooke's Law . . . . .	6
1.5	Transformation of Elastic Stiffnesses Under Rotation of Coordinate System Axes . . . . .	8
1.6	Engineering Constants . . . . .	10
1.7	Plane Stress . . . . .	13
1.8	Ply and Laminate Notation . . . . .	14
<b>Chapter 2</b>	<b>Equations of a Laminated Anisotropic Plate</b>	<b>17</b>
2.1	Basic Assumptions . . . . .	17
2.2	Strain-Displacement Relations . . . . .	18
2.3	Equations of Motion . . . . .	19
2.4	Constitutive Equations . . . . .	23
2.5	Equations of Motion in Terms of Displacements . . . . .	26
2.6	Governing Equations in Terms of a Stress Function and Transverse Displacement . . . . .	32
2.7	Stability of Laminated Plates . . . . .	36
2.8	Boundary Conditions . . . . .	39
<b>Chapter 3</b>	<b>Energy Formulation of Governing Equations</b>	<b>41</b>
3.1	Introduction . . . . .	41
3.2	Strain Energy of a Laminated Plate . . . . .	41
3.3	Kinetic Energy of a Laminated Plate . . . . .	44
3.4	Potential Energy of External Loads . . . . .	45

	3.5	Governing Equations and Natural Boundary Conditions	46
	3.6	The Ritz Method	51
	3.7	The Galerkin Method	53
	3.8	Convergence of the Ritz and Galerkin Methods	56
<b>Chapter 4</b>		<b>One-Dimensional Analysis of Laminated Plates</b>	<b>59</b>
	4.1	Introduction	59
	4.2	Cylindrical Bending	59
	4.3	Buckling and Free-Vibration Under Cylindrical Bending	65
	4.4	Plate Aspect Ratio and Cylindrical Bending	68
	4.5	Bending Analysis of Laminated Beams	68
	4.6	Bending of Laminated Beams Under Concentrated Loads	74
		Buckling and Free-Vibrations of Laminated Beams	82
<b>Chapter 5</b>		<b>Specially Orthotropic Plates</b>	<b>87</b>
	5.1	Introduction	87
	5.2	Bending of Simply-Supported Rectangular Plates	87
	5.3	Bending of Rectangular Plates with Two Simply-Supported Edges	92
	5.4	Bending of Clamped Rectangular Plates	97
	5.5	Stability of Simply-Supported Rectangular Plates Under Uniform Compression	103
		Stability of Rectangular Plates with Two Simply-Supported Edges	108
		Stability of Simply-Supported Rectangular Plates Under Shear Load	112
		Stability of Clamped Rectangular Plates Under Shear Load	114
	5.9	Stability of an Infinite Strip Under Shear Loading	118
	5.10	Free-Vibration of Simply-Supported Rectangular Plates	122
		Free-Vibration of Rectangular Plates with Clamped or Simply-Supported Edges	125
<b>Chapter 6</b>		<b>Midplane Symmetric Laminates</b>	<b>133</b>
	6.1	Introduction	133
	6.2	Bending of Simply-Supported Rectangular Plates	133
	6.3	Bending of Clamped Rectangular Plates	143

	6.4	Stability of an Infinite Strip Under Compression or Shear	147
	6.5	Stability of Simply-Supported Rectangular Plates . .	151
	6.6	Stability of Uniform Rectangular Plates by the . . . . Ritz Method	156
	6.7	Stability of Nonuniform Rectangular Plates . . . . .	162
	6.8	Free-Vibration of Rectangular Anisotropic Plates . .	166
<b>Chapter 7</b>		<b>General Laminated Plates . . . . .</b>	<b>177</b>
	7.1	Introduction . . . . .	177
	7.2	Bending of Rectangular Cross-Ply Plates . . . . .	177
	7.3	Bending of Rectangular Angle-Ply Plates . . . . .	182
	7.4	Bending of Elliptic Cross-Ply Plates . . . . .	185
	7.5	Stability of a Rectangular Angle-Ply Plate Under . . Uniform Biaxial Compression	188
	7.6	Stability of a Cross-Ply Plate Under Uniform . . . . Shear Load	192
	7.7	Free-Vibration of Unsymmetrical Laminated Plates .	197
	7.8	The Reduced Bending Stiffness Approximation . . .	203
<b>Chapter 8</b>		<b>Expansional Strain Effects in Laminated . . . . .</b>	<b>209</b>
	8.1	Introduction . . . . .	209
	8.2	Constitutive Equations . . . . .	209
	8.3	Governing Equations . . . . .	215
	8.4	Strain Energy . . . . .	216
	8.5	Midplane Symmetric Laminates . . . . .	218
	8.6	Bending of Unsymmetric Angle-Ply Laminates . . .	221
	8.7	Thermal Buckling . . . . .	225
	8.8	Effect of Swelling . . . . .	231
<b>Chapter 9</b>		<b>Laminated Cylindrical Plates . . . . .</b>	<b>235</b>
	9.1	Introduction . . . . .	235
	9.2	Constitutive Equations . . . . .	235
	9.3	Governing Equations . . . . .	239
	9.4	Simply-Supported Orthotropic Plates . . . . .	247
	9.5	Stability of Simply-Supported Plates Under . . . . Combined Loading	254
<b>Chapter 10</b>		<b>Shear Deformation in Laminated Plates . . . . .</b>	<b>263</b>
	10.1	Limitations of Laminated Plate Theory . . . . .	263
	10.2	Constitutive Equations . . . . .	263

10.3	Governing Equations . . . . .	265
10.4	Determination of the $k$ Parameter . . . . .	270
10.5	Cylindrical Bending of Orthotropic Laminates . . . . .	273
10.6	Bending of Laminated Beams . . . . .	282
10.7	Bending and Free-Vibration of Angle-Ply . . . . .	290
	Rectangular Plates	
10.8	Analysis of Sandwich Plates . . . . .	295
10.9	Cylindrical Bending of Sandwich Plates . . . . .	302
10.10	Comparison of Sandwich Plate Analysis . . . . .	307
	to Exact Theory	
<b>Chapter 11</b>	<b>Free-Edge Effects and Higher Order . . . . .</b>	<b>313</b>
	<b>Laminated Plate Theory</b>	
11.1	Introduction . . . . .	313
11.2	Free-Edge Effects in Laminated Plates . . . . .	313
11.3	A Thickness-Stretch Deformation Mode in . . . . .	315
	Laminated Plates	
	Calculation of Interlaminar Normal Stress in a . . . . .	319
	Bidirectional Laminate	
	Comparison to Exact Theory . . . . .	327
<b>Appendix:</b>	<b>Laminated Plate Calculations . . . . .</b>	<b>329</b>
	<b>(LAMPCAL—Users' Guide)</b>	
Index		339