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CONTENTS

COMPARISON OF FRICTION FACTOR EQUATIONS FOR NON-NEWTONIAN FLUIDS IN PIPE FLOW[1]

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ABSTRACT

The pressure drop, due to friction, in pipe flow of non-Newtonian fluids can be estimated if the friction factor is known. For laminar flow, the friction factor for power law, Bingham plastic and Herschel-Bulkely fluids can be obtained from a single theoretical relationship. However, a number of different relationships have been developed for turbulent flow. This paper presents a summary and comparison of these relationships. Friction factor predictions were found to differ significantly depending on the flow behavior index, Reynolds number and Hedstrom number. Generally, the spread of predictions increased as the fluid deviated from Newtonian behavior. Significant errors can be made when using relationships based on fluids without a yield stress to predict friction factors for fluids having a yield stress.

INTRODUCTION

Friction factors for non-Newtonian fluids flowing in tubes are required in the mechanical energy balance equation to determine energy losses (Garcia and Steffe 1986; Steffe and Morgan 1986). Friction losses for an incompressible fluid moving in a section of straight pipe or tubing may be written in terms of a pressure loss:

$$\frac{\Delta P_F}{\rho} = \frac{2fLv^2}{D} \qquad (1)$$

The friction factor for laminar flow of Newtonian, power law, Bingham plastic, and Herschel-Bulkley fluids can be obtained from a single relationship. However, numerous correlations exist to estimate the friction factor of these fluids under turbulent flow conditions. Unlike the equations for Newtonian fluids, the predictions for non-Newtonian fluids may differ greatly depending on the relationship used (Heywood and Cheng 1981). The purpose of this article is to summarize and compare friction factor equations for non-Newtonian fluids flowing in pipes.

REVIEW OF LITERATURE

Fluid Models

The flow behavior of many fluid foods and other materials may be described by the Herschel-Bulkley (H-B) model (Herschel and Bulkley 1926):

$$\tau = \tau_0 + K\dot{\gamma}^n \qquad (2)$$

This model simplifies to the power law model when $\tau_0 = 0$:

$$\tau = K\dot{\gamma}^n \qquad (3)$$

A power law fluid is called pseudoplastic when $0 < n < 1$ and dilatant when $n > 1$. Equation (2) reduces to the Bingham plastic model when $n = 1$ and $K = \eta$:

$$\tau = \tau_0 + \eta\dot{\gamma} \qquad (4)$$

Newtonian fluids are described by Eq. (2) when $\tau_0 = 0$, $n = 1$ and $K = \mu$:

$$\tau = \mu\dot{\gamma} \qquad (5)$$

Laminar Flow

The friction factor for the laminar flow of Herschel-Bulkley fluids was presented by Hanks (1978):

$$f = \frac{16}{\psi Re} \qquad (6)$$

in which Re, the generalized Reynolds number, is defined as

$$Re = \frac{D^n v^{(2-n)} \rho}{8^{(n-1)} K} \left(\frac{4n}{1 + 3n} \right)^n \qquad (7)$$

and

$$\psi = (1 + 3n)^n (1 - \xi_0)^{1+n} \left[\frac{(1 - \xi_0)^2}{(1 + 3n)} + \frac{2\xi_0(1 - \xi_0)}{(1 + 2n)} + \frac{\xi_0^2}{(1 + n)} \right]^n \qquad (8)$$

where ξ_0 is the dimensionless unsheared plug radius:

$$\xi_0 = \frac{\tau_0}{\tau_w} = \frac{\tau_0}{(D \Delta P_f / 4L)} = \frac{\tau_0}{(\frac{1}{2} f \rho v^2)} \qquad (9)$$

ξ_0 can also be written as an implicit function of Re and a generalization of the Hedstrom number (He) as (Hanks 1978):

$$Re = 2He \left(\frac{n}{1 + 3n} \right)^2 \left(\frac{\psi}{\xi_0} \right)^{\frac{2-n}{n}} \qquad (10)$$

where

$$He = \frac{D^2 \rho}{K} \left(\frac{\tau_0}{K} \right)^{\frac{2-n}{n}} \qquad (11)$$

For power law and Newtonian fluids, the friction factor can be estimated directly from Eq. (6) since $\xi_0 = 0$ and $\psi = 1$ when $\tau_0 = 0$. For Bingham plastic and H-B fluids, however, ξ_0 must be calculated through iteration of Eq. (10). The range of ξ_0 for laminar flow is $1 > \xi_0 > \xi_{oc}$ where ξ_{oc} is the laminar-turbulent transition value of ξ_0. Once ξ_0 is known, the friction factor is calculated from Eq. (6), (7) and (8). ξ_{oc} will be discussed in more detail later in the paper.

Turbulent Flow Equations for the Power Law Model

Probably the best known correlation for power law fluids is the semi-theoretical equation presented by Dodge and Metzner (1959):

$$\frac{1}{\sqrt{f}} = \frac{4}{n^{0.75}} \log [\text{Re } f^{1-(n/2)}] - \frac{0.4}{n^{1.2}} \tag{12}$$

The authors reported excellent agreement between predicted and experimental friction factors for aqueous Carbopol solutions with flow behavior indexes ranging from 0.36 to 0.73 and generalized Reynolds number from 2900 to 36,000.

Shaver and Merrill (1959) developed a correlation based on experimental results with sodium carboxymethylcellulose (CMC), alginate and aqueous Carbopol solutions with flow behavior indexes ranging from 0.53 to 1.0 and generalized Reynolds number from 7540 to 17,000:

$$f = \frac{0.0791}{n^5 \text{Re}^p} \tag{13}$$

where

$$p = 2.63 / 10.5^n \tag{14}$$

Equations (13) and (14) give friction factors below values predicted by the Dodge and Metzner (1959) relationship [Eq. (12)]. This discrepancy has been attributed to the viscoelastic effect of CMC solutions in the data of Shaver and Merrill (1959) (Skelland 1967; Govier and Axis 1972).

Tomita (1959), on the basis of similarity considerations, derived expressions for the friction factor and Reynolds number for power law fluids in laminar flow. These can be written in terms of the Fanning friction factor and generalized Reynolds number (Govier and Axis 1972):

$$f_{PT} = \left(\frac{4 + 8n}{3 + 9n} \right) f \tag{15}$$

and

$$\text{Re}_{PT} = \left(\frac{3 + 9n}{4 + 8n} \right) \text{Re} \tag{16}$$

Based on Prandtl's mixing length theory, Tomita (1959) obtained a relationship between f_{PT} and Re_{PT} for turbulent flow:

$$\frac{1}{\sqrt{f_{PT}}} = 4\log \left[Re_{PT} \sqrt{f_{PT}} \right] - 0.4 \qquad (17)$$

Equation (17) was verified with starch pastes and lime slurries with flow behavior indexes ranging from 0.178 to 0.952 and Re_{PT} from 1.5×10^3 to 3×10^4.

Thomas (1960a) suggested different coefficients for the Dodge and Metzner relationship as

$$\frac{1}{\sqrt{f}} = \frac{4}{n} \log \left[Re\ f^{1-(n/2)} \right] - \frac{0.4}{n} \qquad (18)$$

Based on the integrated mean velocity of the turbulent core, Clapp (1961) developed a friction factor relationship and confirmed the equation with experimental data of Carbopol solutions having flow behavior indexes ranging from 0.698 to 0.813 over a Reynolds number range of 5480 to 42,800. His equation can be considered a special case of the relationship developed by Torrance (1963) for H-B fluids which will be presented later. Clapp (1961) compared his results with the relationship of Dodge and Metzner (1959) and found that his equation gave a beter representation of experimental data than Eq. (12). Results for Reynolds numbers below 15000 were similar; however, Eq. (12) predicts higher friction factors than Clapp's equation for higher Reynolds numbers. In addition, Clapp's relationship is confirmed for higher Reynolds numbers but Dodge and Metzner (1959) had few data points above 20,000.

Kemblowski and Kolodziejski (1973) derived an empirical equation based on experimental data of aqueous suspensions of keolin with flow behavior indexes ranging from 0.14 to 0.83 and generalized Reynolds number from 2680 to 98,600. These authors found that the relationship of Dodge and Metzner (1959) under predicted their experimental results. They observed that the friction factor curves for non-Newtonian fluids were below the Newtonian curves for Reynolds numbers greater than $Re_{tc} = 3000$ but approached the Newtonian values as the Reynolds numbers increased. They defined this as the transitional flow region with the upper Reynolds number given by

$$Re_t = \frac{3.16 \times 10^4}{n^{0.435}} \qquad (19)$$

For $Re_{tc} < Re < Re_t$, their relationship for the friction factor is

$$f = \frac{E\ A^{1000/Re}}{4Re^m} \tag{20}$$

where

$$A = \exp\left[\frac{0.572(1 - n^{4.2})}{n^{0.435}}\right] \tag{21}$$

$$E = 8.9 \times 10^{-3}\exp(3.57n^2) \tag{22}$$

and

$$m = 0.314n^{2.3} - 0.064 \tag{23}$$

For $Re > Re_t$, they used a Blasius equation for Newtonian fluids:

$$f = \frac{0.0791}{Re^{0.25}} \tag{24}$$

Hanks and Ricks (1975) derived a friction factor relationship for the flow of power law fluids under transitional and turbulent flow conditions. Their model correlated well with the experimental data from Dodge and Metzner (1959), Shaver and Merrill (1959) and Clapp (1961). The relationship is a special case of the model of Hanks (1978) for H-B fluids which will be presented later.

Szilas et al. (1981) analytically determined a friction factor relationship which included the effect of relative roughness of the pipe. They reported their relationship to have the best fit compared to the relationship of Dodge and Metzner (1959), Shaver and Merrill (1959), Tomita (1959) and Clapp (1961) for experimental results with crude oil flowing with a generalized Reynolds number ranging from 10^4 to 10^5, and flow behavior indexes of 0.6 and 0.9. Their relationship predicts increased friction factors as the relative roughness ratio increases. For smooth walls, their friction factor relationship is

$$\frac{1}{\sqrt{f}} = \frac{4}{n} \log \left[Re(4f)^{1-(n/2)} \right] + 1.51^{1/n} \left(\frac{1.414}{n} + 4.24 \right)$$

$$- \frac{8.03}{n} - 2.114 \tag{25}$$

Turbulent Flow for the Bingham Plastic Model

Tomita (1959) also deduced expressions for the friction factor and Reynolds number corresponding to the laminar flow of Bingham plastic fluids. These can be expressed in terms of the Fanning friction factor and generalized Reynolds number, respectively, using (Govier and Aziz 1972)

$$f_{BT} = \frac{f}{1 - \xi_0} \tag{26}$$

and

$$Re_{BT} = (1 - \xi_0)(1 - (4/3)\xi_0 + (1/3)\xi_0^4)Re \tag{27}$$

Tomita applied the Prandtl mixing length concept to obtain a relationship between f_{BT} and Re_{BT} for turbulent flow:

$$\frac{1}{\sqrt{f_{BT}}} = 4\log \left[Re_{BT} \sqrt{f_{BT}} \right] - 0.4 \tag{28}$$

ξ_0 in Eq. (26) and (27) is defined by Eq. (9) and is estimated as if the flow was laminar, i.e., from Eq. (10) with $n = 1$. Equation (28) was confirmed with slurries over a Re_{BT} range of 2×10^3 to 1×10^5 and a He range [as defined by Eq. (11)] of 1.46×10^4 to 5.51×10^5.

Dodge and Metzner (1959) suggested that their relationship for power law fluids [Eq. (12)] could be used as an approximation to predict friction factors for Bingham plastic fluids if n and K are placed by n' and K', respectively. The latter parameters are obtained from a logarithmic plot of τ_w versus $8v/D$ where n' is the slope of the line and K' is the intercept on the ordinate at the point where $8v/D$ equal unity. n' and K' should be evaluated at the wall shear stress for the prevailing turbulent flow conditions. Dodge and Metzner (1959) successfully correlated turbulent flow data for clay suspensions by this technique. However, the validity of this approach has been questioned by several authors (Hanks and Ricks 1975;

Govier and Axis 1972). Heywood and Cheng (1981) presented an assessment of this procedure as well as other methods to estimate friction factors.

Thomas (1960b, 1962, 1963) developed a Blasius type equation based on the turbulent flow of Bingham plastics: flocculated suspensions of kaolin, titanium dioxide and thorium oxide in water. His experimental data covers a He number range of 2.9×10^3 to 8.3×10^5 and results showed the friction factors for these suspensions to be below those for Newtonian fluids. Friction factors approached the Newtonian values as the Reynolds number increased for yield stresses below 24.0 Pa but tended to diverge from the Newtonian values for yield stress values greater than 24.0 Pa. The relationship of Thomas cannot be written in terms of the He number for the purpose of comparison; hence, it will not be considered further.

A semi-theoretical analysis for Bingham plastic fluids was presented by Hanks and Dadia (1971). Their relationship can be considered a special case of the analysis of Hanks (1978). Kenchington (1974) found that the relationship of Hanks and Dadia (1971) predicted his experimental results better than the correlations of Dodge and Metzner (1959), Tomita (1959) and Kemblowski and Kolodziejski (1973). Darby and Melson (1981) developed an expression to approximate the friction factor relationship of Hanks and Dadia (1971).

Turbulent Flow for the H-B Model

Torrance (1963), following the method of analysis of Clapp (1961), developed a relationship for H-B fluids. Using Clapp's constant, he obtained

$$
\frac{1}{\sqrt{f}} = 0.45 - \frac{2.75}{n} + \frac{1.97}{n} \ln(1 - \xi_0) +
$$
$$
\frac{1.97}{n} \ln \left[Re \left(\frac{1 + 3n}{4n} \right)^n f^{1-(n/2)} \right] \tag{29}
$$

where Re and ξ_0 are given by Eq. (7) and (9), respectively.

Combining Eq. (7), (9) and (11), ξ_0 can be written as

$$
\xi_0 = \frac{16(2He)^{n/(2-n)} \left(\dfrac{n}{1 + 3n} \right)^{2n/(2-n)}}{Re^{2/(2-n)}f} \tag{30}
$$

Then Eq. (29) and (30) give f as a function of n, Re and He. Notice that Eq. (30) is only a manipulation of dimensionless numbers and is valid

for any flow conditions. If f is substituted into Eq. (30) using Eq. (6), Eq. (10) for laminar flow is obtained.

Equation (29) reduces to the relationship of Clapp (1961) for power law fluids for $\xi_0 = 0$. Notice also that this equation reduces to the Bingham plastic model for $n = 1$. Equation (29) has not been verified experimentally for Bingham plastic or H-B fluids. Torrance (1963) also developed an equation to account for turbulent flow in rough pipes and further information can be found in Torrance (1963) and Govier and Aziz (1972).

The Dodge and Metzner (1959) approach for Bingham plastic fluids described in the previous section can also be used to approximate friction factor predictions for H-B fluids in turbulent flow. Heywood (1980) suggested that n' could be approximated as $n' = d \ln(\tau_0 + K\dot{\gamma}_w^n)/d \ln(\dot{\gamma}_w)$ when only rotational viscometric data was available.

Probably the most comprehensive analysis for H-B fluids in turbulent flow has been presented by Hanks (1978). Unlike the relationships presented up to now [with the exception of Hanks and Ricks (1975) and Hanks and Dadia (1971)], Hanks' analysis deals with transitional flow and includes the laminar-turbulent transition criterion developed by Hanks and Ricks (1974). As stated before, this model reduces to the models of Hanks and Ricks (1975) for power law fluids ($\xi_0 = 0$) and Hanks and Dadia (1971) (with some improved modifications) for Bingham plastic fluids. Even though the model of Hanks (1978) has not been experimental verified, it has been successfully confirmed for the special cases of Bingham plastic, power law and Newtonian fluids.

The laminar-turbulent transition criterion developed by Hanks and Ricks (1974) is

$$\text{Re}_c = \frac{6464 n \psi_c^{(2/n)-1}(2 + n)^{(2+n)/(1+n)}}{(1 + 3n)^2(1 - \xi_{oc})^{(2/n)+1}} \tag{31}$$

where Re_c is the critical Reynolds number, the point where laminar flow ends. ξ_{oc} is the laminar-turbulent transition value of ξ_0 and is given implicity by

$$\text{He} = \frac{3232(2 + n)^{(2+n)/(1+n)}\xi_{oc}^{(2/n)-1}}{n(1 - \xi_{oc})^{(2/n)+1}} \tag{32}$$

where ψ_c is determined from Eq. (7) by replacing ξ_0 with ξ_{oc}. For $\text{Re} < \text{Re}_c$, the flow is laminar and f is estimated from Eq. (6) through (11) with $1 > \xi_0 > \xi_{oc}$. For $\text{Re} = \text{Re}_c$, the flow is at the critical point and

the critical friction factor (f_c) is estimated from Eq. (6) with $\psi = \psi_c$ and $Re = Re_c$. For transitional and turbulent flow, $Re > Re_c$, the friction factor is obtained from Hanks (1978) relationships which can be expressed in terms of the generalized Reynolds number:

$$Re = (1 - \xi_0)^{(2-n)/n} \left(\frac{n}{1 + 3n} \right)^n R^2 \left[\int_{\xi_0}^1 \xi^2 \varsigma(\xi)d\xi \right]^{2-n} \tag{33}$$

where R is a working parameter:

$$R = \left(\frac{1 + 3n}{n} \right) [Re(f/16)^{(2-n)/n}]^{1/n} \tag{34}$$

The dimensionless shear rate (ς) is an implicit function of the dimensionless radial coordinate (ξ) found through the time average momentum flux expression written in dimensionless form as

$$\xi = \xi_0 + (1 - \xi_0)\varsigma^n + \frac{R^2}{8}(1 - \xi_0)^{2/n}\lambda^2\varsigma^2 \tag{35}$$

where λ is a modified Prandtl's mixing length parameter (in dimensionless form) which takes into account the effect of transitional flow characteristics, the viscous damping effect of the wall as well as the radial variation of shear stress. The expression for λ is

$$\lambda = 0.36(1 - \xi) \{1 - \exp[-\phi(1 - \xi)]\} \tag{36}$$

where

$$\phi = \frac{R - R_c}{\sqrt{8} \, B} \tag{37}$$

B is an empirical parameter given for H-B fluids as

$$B = \frac{22}{n} \left(1 + \frac{0.00352He}{(1 + 0.000504He)^2} \right) \tag{38}$$

R_c is the laminar-turbulent transition value of R obtained from Equation (34) by replacing f and Re with f_c and Re_c, respectively. The relationship between f, ξ_o, Re and He is given by Eq. (30), and can be rewritten in terms of R [Eq. (34)]:

$$R^2 = \frac{2He}{\xi_o^{(2-n)/n}} \tag{39}$$

For turbulent flow, $\xi_o < \xi_{oc}$.

The major setback of Hank's method is that it involves a numerical integration and a number of iterations. Hanks (1978) provided a procedure to develop f versus Re diagrams with n and He as parameters. Garcia (1985) developed a computer program from which f can be estimated given the fluid properties, mass flow rate and pipe diameter (i.e. given n, He and Re).

Further discussion of the relationships presented in this article may be found in papers by Cheng (1970), Cheng (1975), Heywood and Cheng (1981), Kenchington (1974), Govier and Aziz (1972) and Skelland (1967).

RESULTS AND DISCUSSION

Friction factor graphs have been made for the various relationships presented. Figures 1 through 8 show the relationships for turbulent flow for power law fluids (and Newtonian fluids for n = 1). The laminar flow line |Eq. (6) through (11)| is also shown in each figure. As seen, the relationship of Hanks and Ricks (1975) [Fig. (7)] is the only one that deals with the transitional region. The Tomita (1959) relationship [Fig. (3)] predicts small variation of f with n for a given Re, compared to the other equations. The same is true (but to a lesser extent) for the relationship of Kemblowski and Kolodziejski (1973) [Fig. (6)]. Conversely, the correlations of Szilas et al. (1981) [Fig. (8)] and Shaver and Merrill (1959) [Fig. (2)] show a greater variation of f with n, particularly as n decreases. The latter relationship is restricted for n greater than 0.5. Govier and Aziz (1972) suggested that this correlation [Fig. (2)] could not be considered for general uses since it is mostly based on experimental data of CMC solutions, now known to have viscoelastic properties. The prediction of the other power law relationships seem to fall in a closer range but deviated as n decreases and Re increases.

Figures (9), (10) and (13) show the friction factor relationship for laminar and turbulent flow of Bingham plastic fluids. As seen from Fig. (13), Hanks
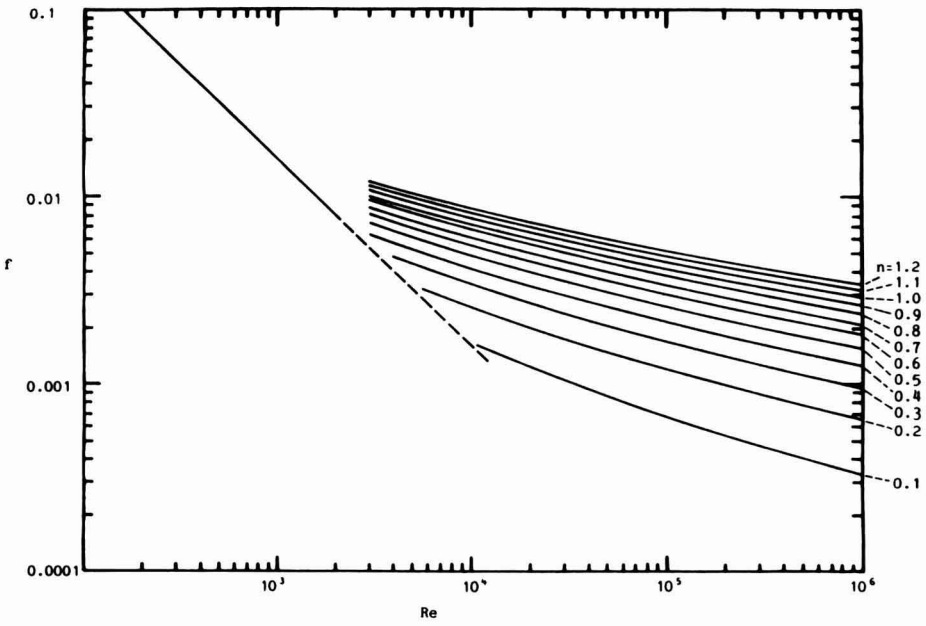
FIG. 1. RELATIONSHIP OF DODGE AND METZNER (1959)
FOR POWER LAW FLUIDS, EQ. (12)
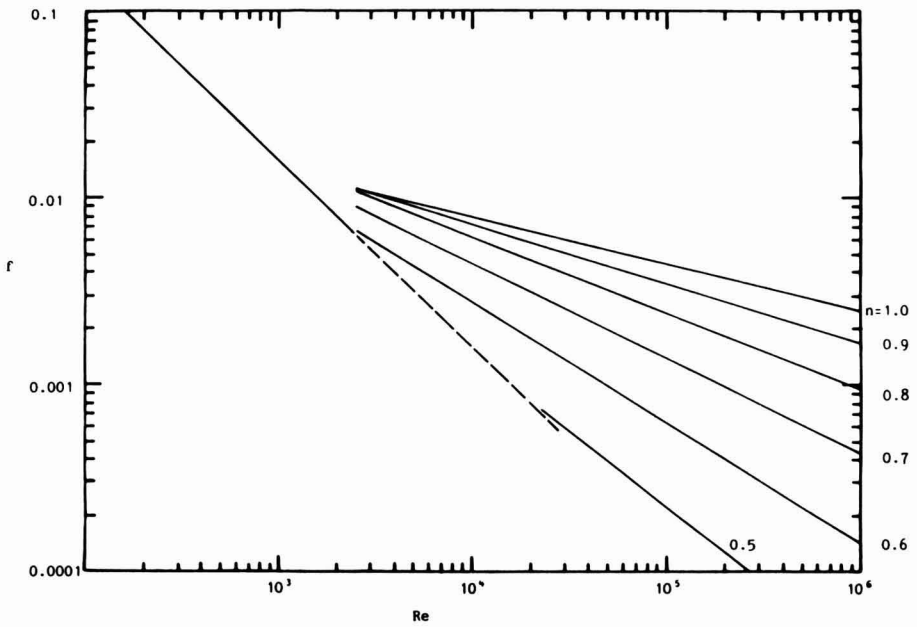
FIG. 2. RELATIONSHIP OF SHAVER AND MERRILL (1959)
FOR POWER LAW FLUIDS, EQ. (13) THROUGH (14)
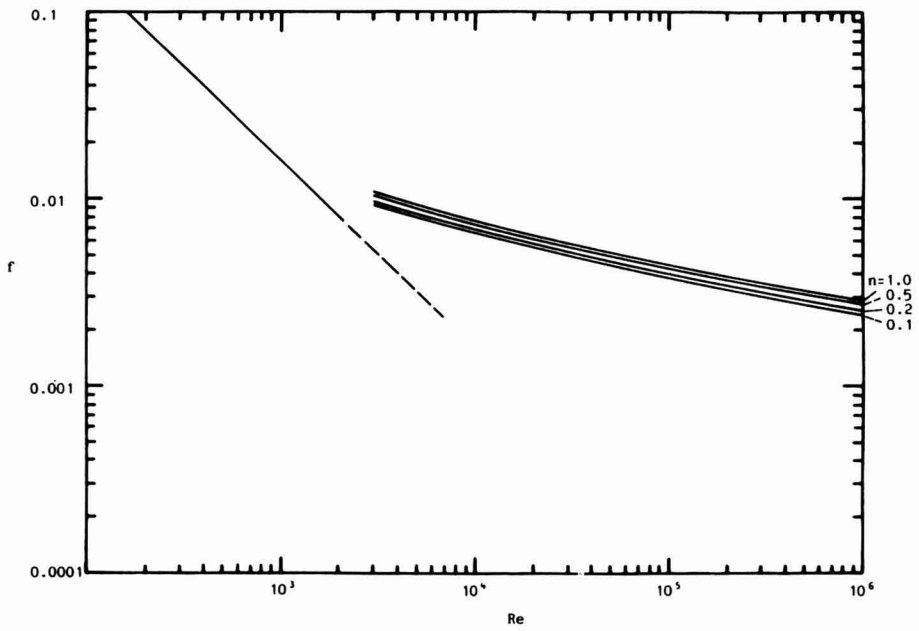
FIG. 3. RELATIONSHIP OF TOMITA (1959) FOR POWER LAW FLUIDS,
EQ. (15) THROUGH (17)
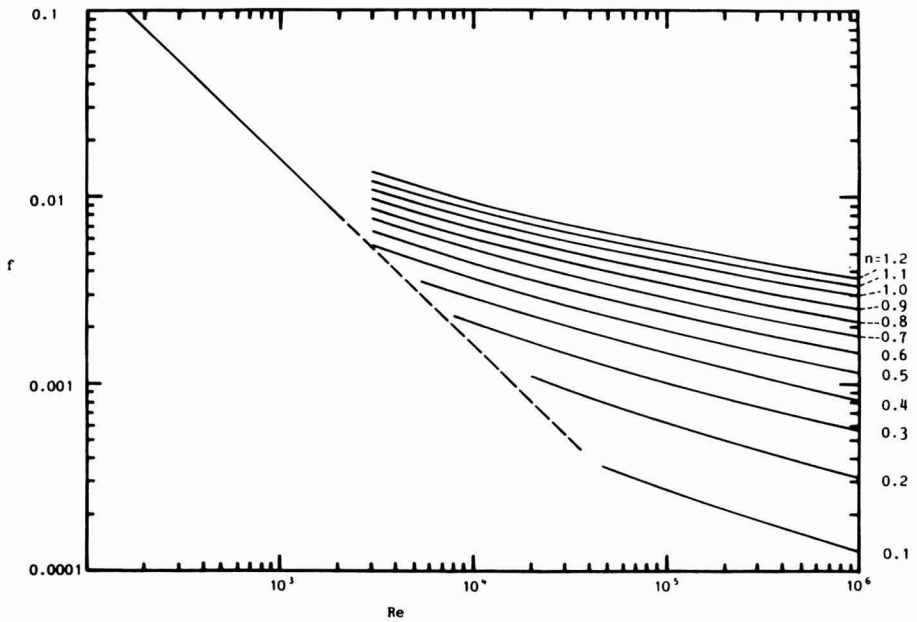
FIG. 4. RELATIONSHIP OF THOMAS (1960) FOR POWER LAW FLUIDS, EQ. (18)

E. J. GARCIA and J. F. STEFFE
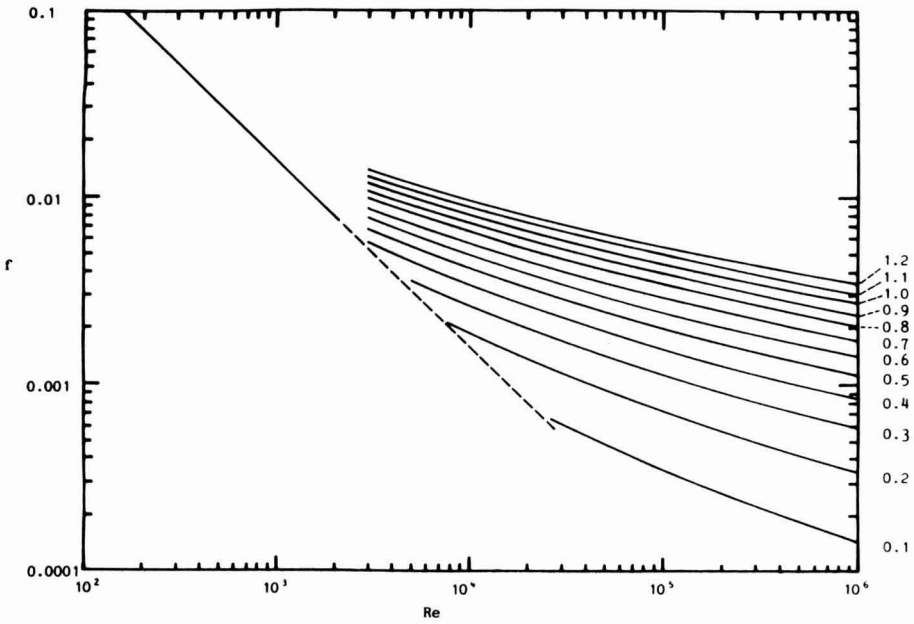
FIG. 5. RELATIONSHIP OF CLAPP (1961) FOR POWER LAW FLUIDS,
EQ. (29) WITH $\xi_o = 0$
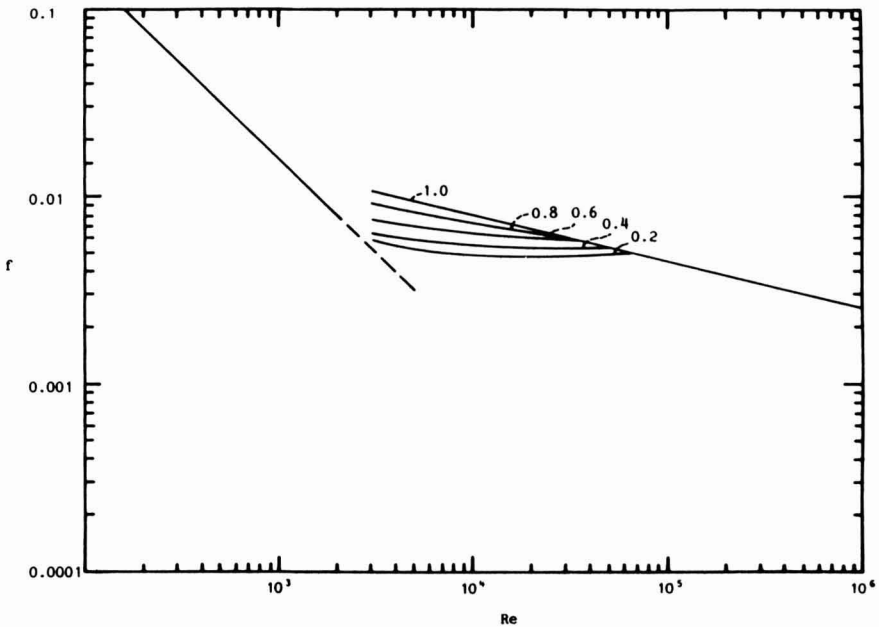
FIG. 6. RELATIONSHIP OF KEMBLOWSKI AND KOLODZIEJSKI (1973)
FOR POWER LAW FLUIDS, EQ. (19) THROUGH (24)
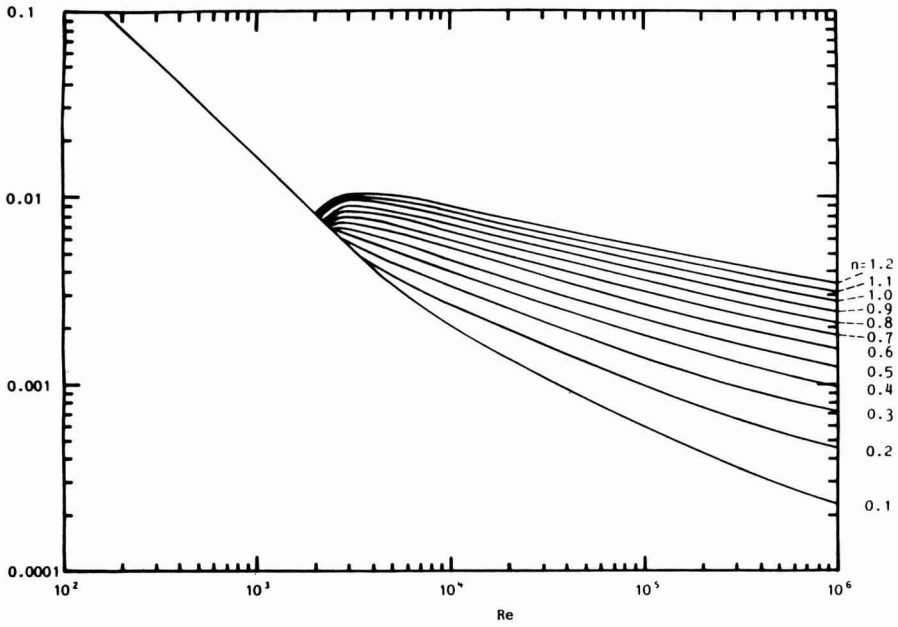
FIG. 7. RELATIONSHIP OF HANKS AND RICKS (1975) FOR POWER LAW
FLUIDS, EQ. (31) THROUGH (39) WITH $\xi_o = 0$ AND $\xi_{oc} = 0$
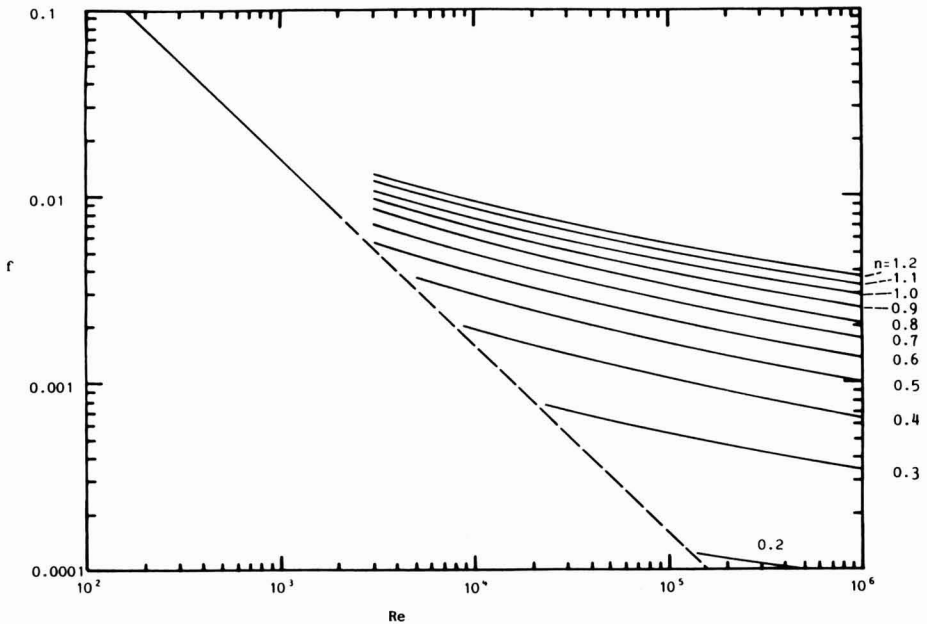
FIG. 8. RELATIONSHIP OF SZILAS *ET AL.* (1981) FOR POWER LAW FLUIDS,
EQ. (25)
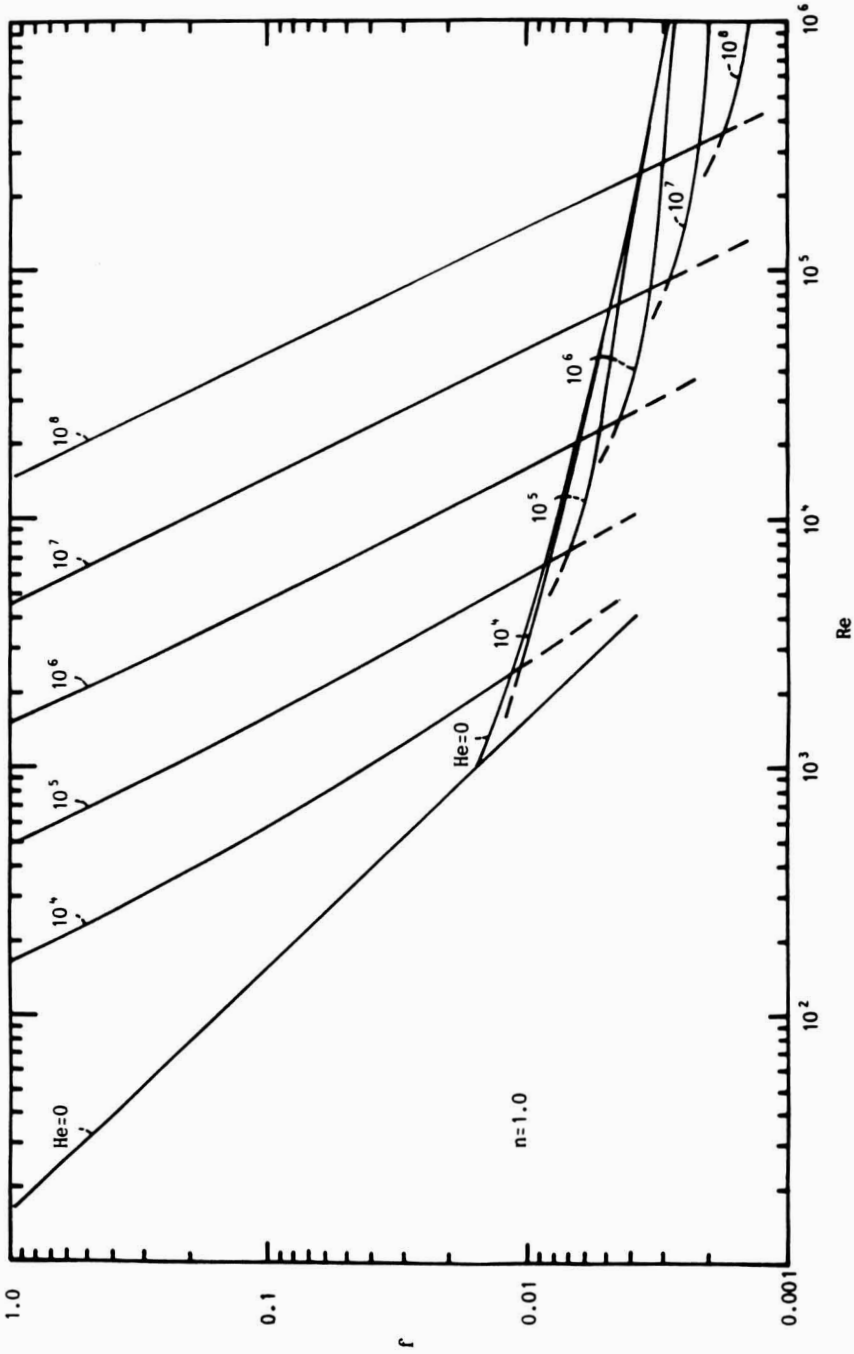
FIG. 9. RELATIONSHIP OF TOMITA (1959) FOR BINGHAM PLASTIC FLUIDS, EQ. (26) THROUGH (28)
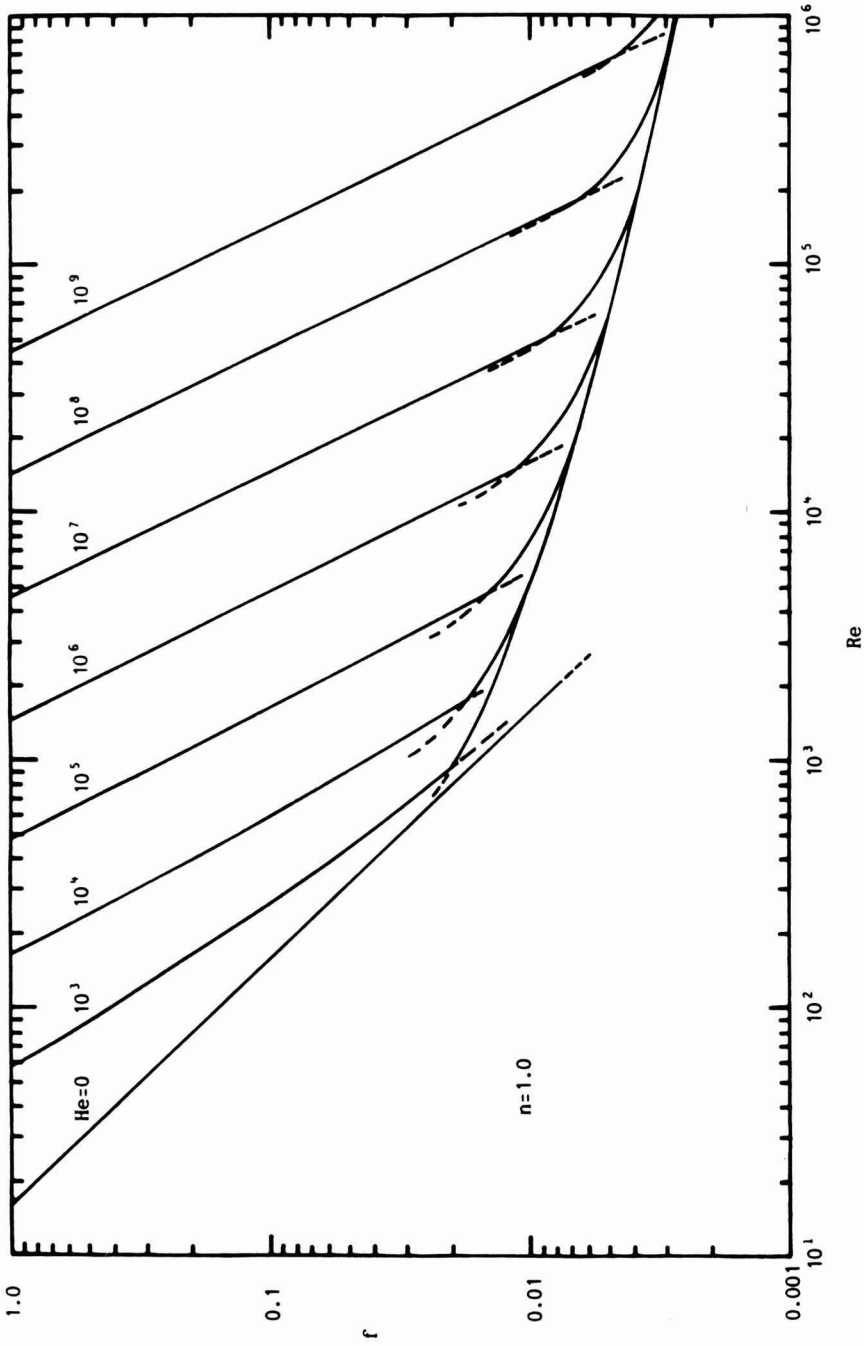
FIG. 10. RELATIONSHIP OF TORRANCE (1963) FOR BINGHAM PLASTIC FLUIDS, EQ. (29) AND (30)

and Dadia (1971) is the only one that deals with the transition from laminar to turbulent flow. This figure shows the variation of Re_c with He (dashed line) obtained from Eq. (31) and (32). The curves to the left of Re_c correspond to the laminar flow region and are obtained from Eq. (6) through (11). For Fig. (9) and (10) the turbulent curves for the different He values are extended until they meet the corresponding laminar flow curve. The interception of these curves could be considered as the laminar-turbulent transition point. As seen from Fig. (9), the friction factors for Bingham plastic fluids predicted by the Tomita (1959) relationship are below the Newtonian values (He = 0). These predictions approach the Newtonian values as Re increases. Conversely, the Torrance (1963) relationship [Fig. (10)] predicts values above the Newtonian numbers. As in the Tomita (1959) curves, the Bingham plastic curves of Torrance (1963) correlation merges with the Newtonian curve as Re increases. The relationship of Hanks and Dadia (1971) [Fig. (13)], on the other hand, predicts friction factors below the Newtonian values for He numbers below 5×10^5, similar values for a He range of 5×10^5 to 1×10^6, and higher values for He numbers greater than 1×10^6. For He $> 10^5$, the Bingham plastic curves eventually merge with the Newtonian curve (black dots) as Re increases. For lower He values, however, the Bingham plastic friction factor predictions stay below the Newtonian values following a curve similar to the Newtonian curve.

Figures (11) and (12) show the Torrance (1963) relationship for H-B fluids with n = 0.5 and n = 0.2, respectively. The laminar flow curves, obtained from Eq. (6) through (11), are also shown and are extended up to the intersection of the Torrance (1963) curves for the various He numbers considered. The Torrance (1963) equation predicts higher friction factors for H-B fluids than the corresponding power law fluid (He = 0, n ≠ 1). The H-B curves also merge with the power law curves as Re increases. Figures 14 and 15 show the friction factor prediction for H-B fluids according to the analysis of Hanks (1978) for n = 0.5 and n = 0.2, respectively. The corresponding power law curves (He = 0) are indicated with black dots. In Fig. (13), the H-B curves fall below their corresponding power law curves for lower He numbers. As n decreases, however, these curves start falling above the power law curves. For n = 0.2, for example, most of the friction factor predictions for H-B fluids are above the corresponding power law predictions for the Re range shown. Notice also that for smaller n and high He values there is an abrupt laminar-turbulent transition. This also occurs at higher n values but low He numbers. For high n and He numbers, very extended transitional regimes are observed. Also the critical Reynolds number increases with increasing He numbers and has a smaller range as n decreases.
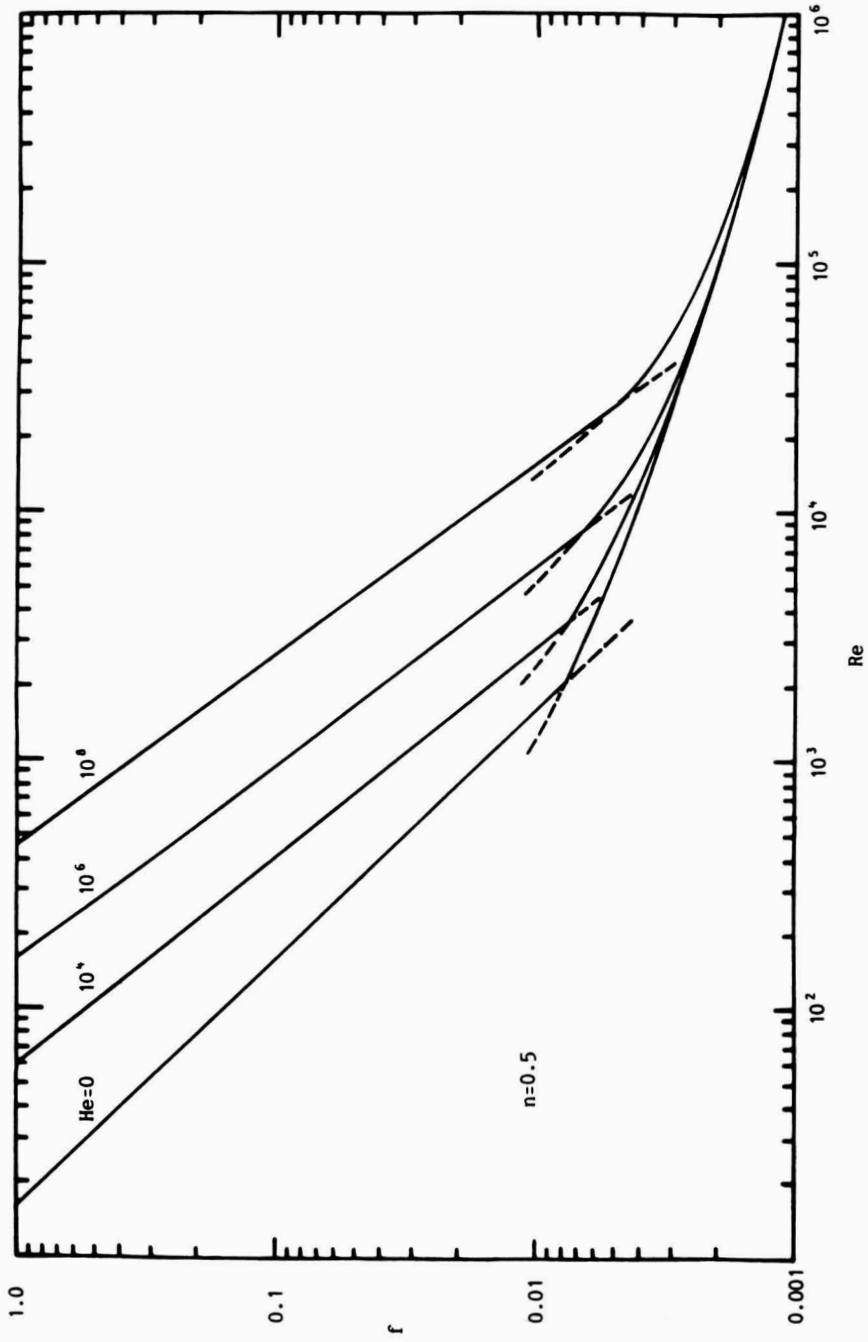
FIG. 11. RELATIONSHIP OF TORRANCE (1963) FOR H-B FLUIDS WITH n = 0.5, EQ. (29) AND (30)
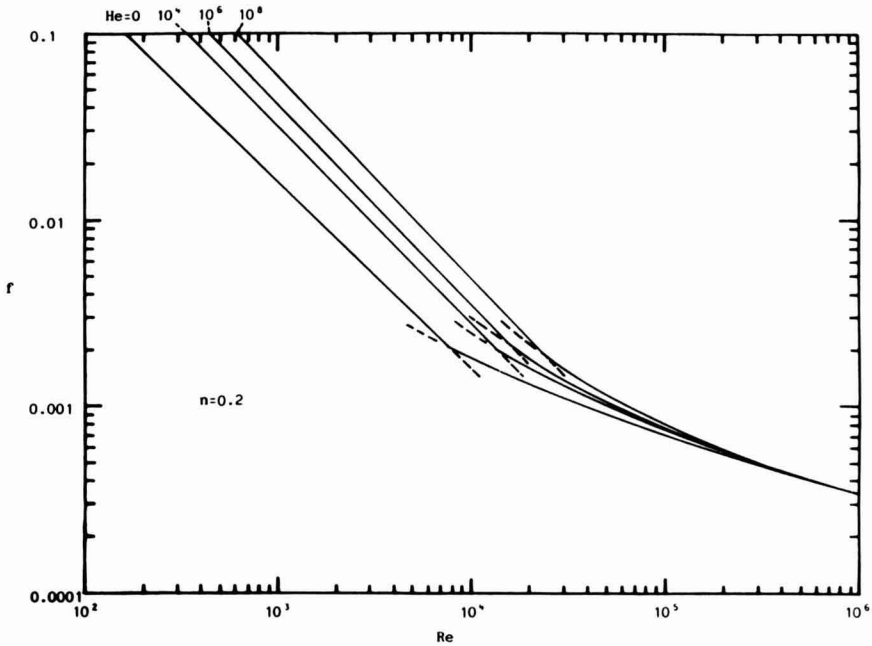
FIG. 12. RELATIONSHIP OF TORRANCE (1963) FOR H-B FLUIDS WITH n = 0.2,
EQ. (29) AND (30)

It is interesting to note that at higher n and low He numbers, variations in He result in small changes in the friction factor. At low n, however, the same variations in He result in greater changes in f. For example, for n = 1.0 and Re = 2 × 10³, a variation of He from 0 to 10³ results in a 8.3% increase in f. The same variation in He (at the same Re number) for n = 0.5 and n = 0.2 results in a 42.15% and 70.36% increase in f, respectively. The opposite is true at higher He numbers. A variation of He from 10³ to 10⁵ at Re = 2 ×10³ result in an increase in f of 685.29% for n = 1.0 and 102.81% for n = 0.5. However, a 26.89% increase in f is obtained for n = 0.2. A similar pattern is found in turbulent flow. These variations also indicate that ignoring the yield stress for H-B or Bingham plastic fluids may lead to severe errors in predicting the friction factor. For example, if the power law curve for n = 0.5 was to be used to predict the friction factor at Re = 10⁴ for a H-B fluid with same flow behavior index, and He = 10³, the friction factor would be over estimated by 44.48% according to the relationship of Hanks (1978). For He = 10⁶, the friction factor would be under estimated by 44.23%.
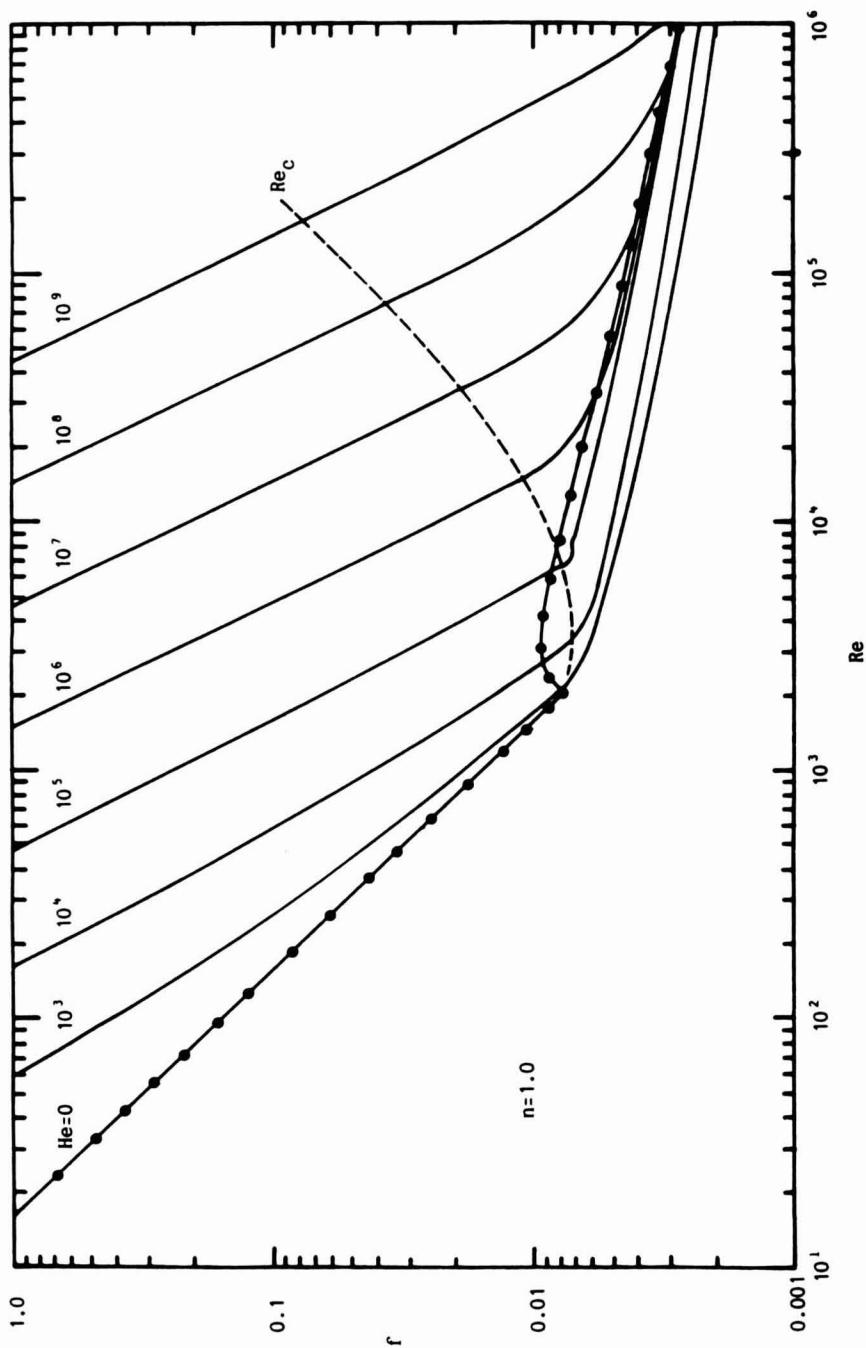
FIG. 13. RELATIONSHIP OF HANKS AND DADIA (1971) FOR BINGHAM PLASTIC FLUIDS
WITH IMPROVED MODIFICATION OF HANKS (1978), EQ. (6) THROUGH (11) AND (31) THROUGH (39) WITH n = 1
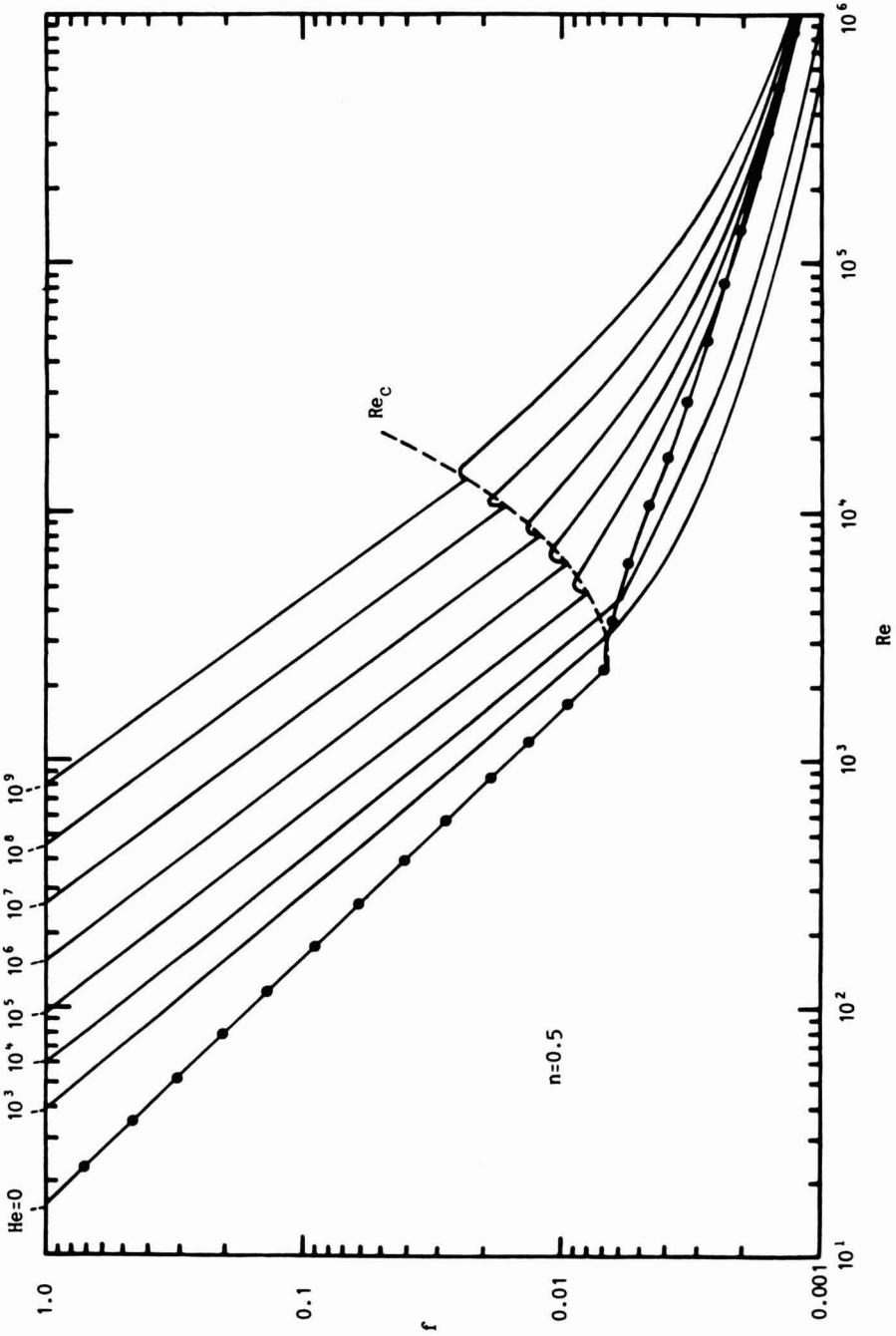
FIG. 14. RELATIONSHIP OF HANKS (1978) FOR H-B FLUIDS WITH n = 0.5, EQ. (6) THROUGH (11) AND (31) THROUGH (39)
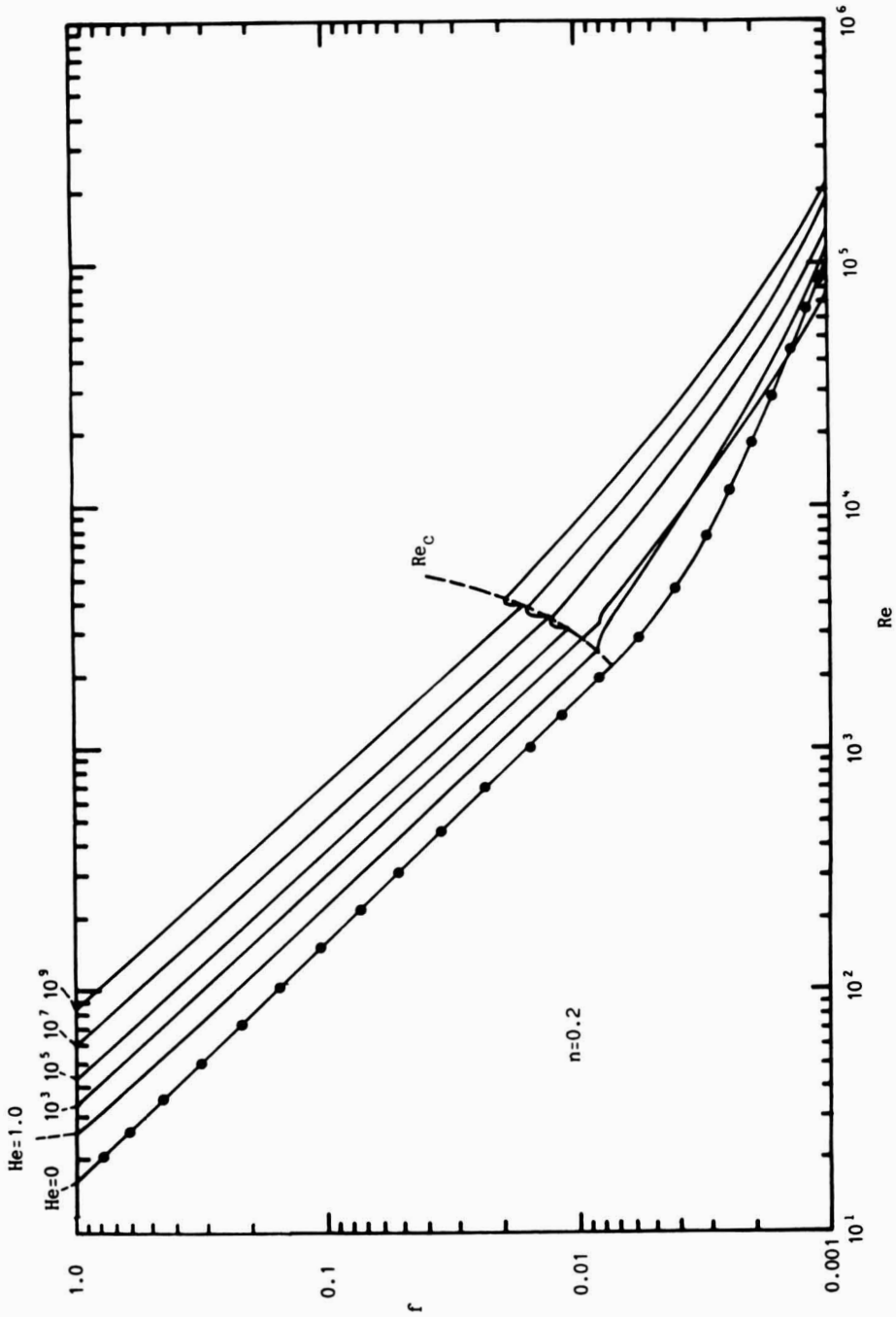
FIG. 15. RELATIONSHIP OF HANKS (1978) FOR H-B FLUIDS WITH n = 0.2, EQ. (6) THROUGH (11) AND (31) THROUGH (39)

To further study the various friction factor relationships, the predictions were compared at various n, Re, and He values. For Newtonian fluids, the variation in the predictions were found to be small. A spread of $\pm$ 2.4% and $\pm$ 1.1% were found for Re = 10^4 and Re = 10^5, respectively. This is to be expected since most of these relationships reduce to well known equations for Newtonian fluids in turbulent flow (Govier and Aziz 1972). However, the spread of the prediction increases as n decreases. For n = 0.5 and Re = 10^4 the spread is $\pm$ 67.5%. If only the relationships of Dodge and Metzner (1959), Thomas (1960a), Clapp (1961), and Hanks and Ricks (1975) are considered, this variation is $\pm$ 14.5% (only the latter relationships will be considered in subsequent analysis of power law fluids). For n = 0.2 at the same Re (10^4), the spread increases to the $\pm$ 32.0%. For power law fluids, the variation of the prediction generally increases with increasing Re. At n = 0.5, for example, the prediction spread is $\pm$ 15.1% and $\pm$ 16.2% for Re = 10^5 and 10^6, respectively. For Bingham plastic and H-B fluids, the spread of predictions generally decreases with increasing Re for a given He and n. For n = 0.5 and He = 10^5, the spread of predictions were $\pm$ 12.6%, $\pm$ 5.1% and $\pm$ 2.9% for Re of 10^4, 10^5 and 10^6, respectively. This trend is due to the fact that most of the Bingham plastic and H-B curves approach the Newtonian and power law curves, respectively, as Re increases. The prediction spread seems to have a minimum for a He number of approximately 10^5 for higher values of n, but seems to increase with increasing He numbers for small n values. With n = 1 and Re = 10^5, for example, the spread of the prediction was found to be $\pm$ 23%, $\pm$ 16%, $\pm$ 3% and $\pm$ 14% for He of 10^3, 10^4, 10^5 and 10^6, respectively. At n = 0.2, however, the spread was found to be $\pm$ 8%, $\pm$ 14%, $\pm$ 22% and $\pm$ 25%, respectively. This is due to the trend of the relationship of Hanks (1978). Much higher differences in predictions are found at higher He numbers.

SUMMARY AND CONCLUSIONS

(1). A summary of friction factor relationships for non-Newtonian fluids has been presented. (2). The friction factor predictions from these relationships were found to differ significantly depending on the n, Re and He numbers. (3). The relationships of Dodge and Metzner (1959), Thomas (1960a), Clapp (1961) and Hanks and Ricks (1975) for power law fluids were found to predict similar friction factors. The analysis of Hanks (1978) was found to be the most comprehensive study for H-B fluids to date; however, more experimental confirmation with a greater range of fluids

is necessary to select the best relationship. (4). The use of power law or Newtonian relationships to predict friction factors for fluids having a yield stress may lead to significant errors.

LIST OF SYMBOLS

A	= parameter in the relationship at Kemblowski and Kolodziejski (1973), Eq. (21)
B	= empirical wall effect parameter in mixing length theory, Eq. (38)
D	= pipe inside diameter, m
E	= parameter in the relationship of Kemblowski and Kolodziejski (1973), Eq. (22)
f	= Fanning friction factor, Eq. (1)
f_{BT}	= Tomita (1959) friction factor for Bingham plastic fluids, Eq. (26)
f_c	= laminar — turbulent transition value of f
f_{PT}	= Tomita (1959) friction factor for power law fluids, Eq. (15)
He	= generalized Hedstrom number, Eq. (11)
K	= consistency coefficient, Pa s^n
L	= pipe length, m
m	= parameter in the relationship of Kemblowski and Kolodziejski (1973), Eq. (23)
n	= flow behavior index, dimensionless
p	= parameter in the relationship of Shaver and Merrill (1959), Eq. (14)
r_w	= radius at inside wall of pipe ($r_w = D/2$), m
R	= turbulent parameter in the relationship of Hanks (1978), Eq. (34)
R_c	= laminar-turbulent transition value of R
Re	= generalized Reynolds number, Eq. (7)
Re_{BT}	= Tomita (1959) Reynolds number for Bingham plastic fluids, Eq. (27)

Re_c = laminar-turbulent transition value of Re, Eq. (31)

Re_{PT} = Tomita (1959) Reynold number for power law fluids, Eq. (16)

Re_t = upper limit Reynolds number demarking the end of transitional flow according to Kemblowski and Kolodziejski (1973) relationship, Eq. (19)

Re_{tc} = Reynolds number demarking the end of laminar flow according to Kemblowski and Kolodziejski (1973) relationship, 3,000.

v = mass average velocity, m/s

Greek Symbols

$\dot{\gamma}$ = rate of shear, s^{-1}

ΔP_f = pressure drop due to friction, Pa

ζ = dimensionless rate of shear

η = plastic viscoisty, Pa s

λ = dimensionless mixing length, Eq. (36)

μ = Newtonian viscosity, Pa s

ξ = dimensionless radial coordinate, τ/τ_w

ξ_o = dimensionless unsheared plug radius, τ_o/τ_w

ξ_{oc} = laminar-turbulent transition value of ξ_o

ρ = fluid density, kg/m^3

τ_o = yield stress, Pa

τ = shear stress, Pa

τ_w = value of τ at r_w

Φ = parameter in mixing length, Eq. (37)

ψ = laminar flow function, Eq. (8)

ψ_c = laminar-turbulent transition value of ψ

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HYDRODYNAMICS AND HEAT TRANSFER IN LIQUID FULL SCRAPED SURFACE HEAT EXCHANGERS — A REVIEW

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ABSTRACT

Theoretical and empirical models pertaining to the hydrodynamics and heat transfer in liquid full scraped surface heat exchangers have been reviewed up to date. In hydrodynamics, various aspects viz. fluid flow, residence time distribution, power requirement are covered. The heat transfer characteristics have been reviewed from stand points of heating, cooling and ultra high temperature applications. The limitations of various models are explained. The logical conclusions and the areas needing further investigations have been delineated.

INTRODUCTION

The transfer of heat into (or from) viscous or heat sensitive liquids is a problem which confronts process engineers in certain industries, specially in food industry. Because of the very wide range of products associated

with this industry, ranging from low viscosity liquids to high viscosity products that can often contain particulate matter, the application of heat exchangers has to be very carefully considered in designing a process plant. To illustrate this, manufacturers of food process equipment have to consider products ranging from liquids such as fruit juices and milk, where the viscosity is little more than that of water, upto very high viscosity products such as fruit concentrates, purees, pie fillings and puddings, the latter often containing food particles. The viscosity of these products also varies at different stages of a heating and cooling process; for example, when the pre-mix, containing a thickening agent such as starch, has initially a low viscosity, heating will cause gelatinization of starch giving rise to a higher viscosity product during final stage of process (Smith 1972). One but by no means the only way of overcoming or reducing the difficulty is to scrape continuously the surface over which the liquid is flowing. In this technique the slow moving layer at the surface which restricts the rate of heat transfer, is removed and at the same time fresh liquid comes into contact with the surface. A vertical liquid full scraped surface heat exchangers (SSHE), is shown in Fig. 1. This technique has been known for many years, but little data on the factors which affect the heat transfer under these conditions have been published. Design has been largely left in the hands of companies engaged in the actual manufacture of scraped surface heat exchangers, who naturally keep detailed operating data as restricted as possible. There has been little opportunity for individuals in the process industries to tackle the design of these exchangers, in the same way as they would size say, a shell and tube exchanger (Bott 1966).

The technical literature is increasing at such a fast rate that it is difficult to keep abreast of significant developments in even a limited area without some help in condensation and interpretation. The aim of this paper is to present review of literature pertaining to hydrodynamics and heat transfer in liquid full SSHEs.

HYDRODYNAMICS

Fluid Flow

Fluid flow in SSHE is very complex, since there are two nonzero components of the velocity field, the axial velocity and the rotational velocity, which generate what is usually called helical flow, where the fluid particles follow a helical or spiral path through the exchanger. The helical flow is usually characterized in terms of the axial Reynolds number, Re and Taylor number, Ta. When a certain value of rotational speed is reached, there appear pairs of counter-rotating vortices and the rotational flow tends

PRODUCT OUTLET

HYDRAULIC DRIVE

MOUNTING CONNECTION
MEDIA

ROTOR SHAFT

INSULATION

HEAT TRANSFER WALL

MEDIA CHANNEL

BLADES

HYDRAULIC DEVICE

FLOATING "O" RING SEAL

MEDIA

PRODUCT INLET

HEAT TRANSFER CYLINDER

BLADE

PRODUCT

ROTOR

HEATING OR COOLING MEDIA
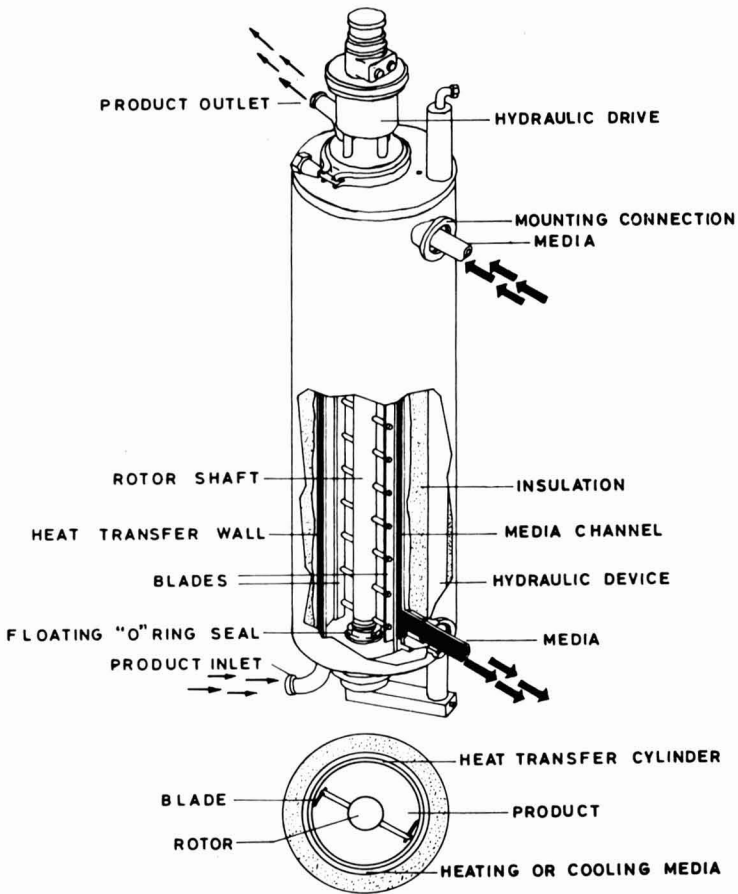
FIG. 1. LIQUID FULL VERTICAL SSHE
Source: Alfa-Level Catalogue

to stabilize. The critical Taylor number, Ta_{cr} (the value of Ta at which vortices will first appear) is a function of axial Reynolds number.

Trommelen and Beek (1971) studied flow phenomenon by suspending polyethene beads in glycerol/water solutions, and operating a glass walled scraped surface heat exchanger with this suspension. It was established that the flow was either in the couette flow regime or in the Taylor vortice regime. The flow in the couette regime was further studied in a perspex model of heat exchanger in which a dye could be injected at different positions. In this way stream line patterns and velocity profiles were determined. The stream lines were concentric circles outside the blade area. The stream lines contract and expand near the blades. The resulting velocity profile is similar to that between two parallel plates, one of which

is moving with a pressure gradient in the direction of the flow. However, above a certain critical rotational Reynold number, large secondary toroidal vortices were formed in pairs with opposite rotation. The diameter of a vortex was approximately equal to the width of annular space. Within the accuracy of the experiments, the critical Reynolds number is equal to that for flow in annulus between smooth rotating cylinders.

Hasoon and Martin (1977) and Simmers and Coney (1979) obtained the functional relationship between axial Reynolds number and critical Taylor number which seem to follow a power law type behavior. The equations are as follows:

When $(D/D_s = 0.955)$ and $300 < Re_a < 600$ $Ta_{cr} = 162.87\ Re_a^{0.916}$ (1)

$\quad\quad\ (D/D_s = 0.9)\quad\quad 400 < Re_a < 2000$ $Ta_{cr} = 41.79\ Re_a^{1.08}$ (2)

$\quad\quad\ (D/D_s = 0.5)\quad\quad 200 < Re_a < 400$ $Ta_{cr} = 106.24\ Re_a^{1.09}$ (3)

Residence Time Distribution

Trommelen and Beek (1971) measured residence time distribution both in the couette and Taylor regime by injecting a pulse of dye near the inlet of heat exchanger and sampling continuously at the outlet. The residence time of the liquid was not uniform. This was partly attributed to differences in velocity along, and in length of stream lines and partly because of dispersion mechanisms due to vortices and a pumping effect.

Cuevas et al. (1980) investigated the residence time distribution in SSHE thermal processor in ultra high temperature (UHT) range (130°C). A step input was used and tracer concentration (heat-stable food color, FD & C 3 MC00903, Stange Co., Chicago, IL, 3.4g/100 gallons of water) moniored by sampling the outlet of the system at every 15 s and measuring the absorbance at 600 nm in a Bausch and Lamb spectronic 20 spectrophotometer. The behavior was observed to be somewhere between ideal laminar flow and plugflow, probably closer to plug flow.

Power Requirement

The work on process agitators has been, for the most part, concerned with fitting dimensionless parameters to experimental data. Dimensional analysis is successful in so far as the power data for any particular process agitator, given complete geometrical similarity, can be correlated for a Newtonian fluid by plotting the conventional power number Po against rotational Reynolds number, Re_R. This correlating method, then accurately reflects the effect of fluid properties.

Generally, below Reynolds number of 20, the curve of Po versus Re_R has a slope of -1 on log-log scale. This regime is called viscous because inertia force are negligible. In the range of $20 < Re_R < 10,000$ the slope gradually changes from -1 to 0. This is transition regime where both viscous and inertial effects are important. Uhl and Voznick (1960) reported the onset of turbulent regime at about $Re_R = 600$. Thus in the range $20 < Re_R < 600$, the flow is laminar. Inertial effects become increasingly important as Re_R increases. In the range $600 < Re_R < 10,000$, the flow becomes turbulent. Above $Re_R = 10,000$ the curve of Po versus Re_R is essentially flat because the power becomes independent of viscosity.

Close clearance agitators are most frequently used with viscous materials. For this reason most of the power data is in the viscous regime ($Re_R < 20$) with a considerable portion extending into transition regime ($20 < Re_R < 10,000$).

Skelland and Leung (1962) investigated the effect of rotational speed and number of blades and presented the data for two row blades in the form of dimensionless correlation.

$$Po^* = 77,500\ Re_R^{-1.27}B^{0.59} \tag{4}$$

Their tests were in the lower transion regime ($90 < Re_R < 242$) even so, the slope of log Po* versus log Re_R was -1.27. Dinglinger (1964) correlated the experimental data in the form:

$$Po = 36,552\ Re_R^{-1.2}$$

$$20 < Re_R < 20,000.$$

It was shown that length of heat exchanger also contributes to power consumption.

Trommelen and Boerema (1966) showed that there is no unique relationship between Po and Re_R; at constant Re_R Po decreases with increasing viscosity. This effect was explained by a temporary increase of the temperature of the liquid between the edge of the scraper blade and tube wall.

In their correlation, the authors did not distinguish the two parts of the power consumption, viz. the dissipation in the bulk of the liquid in the annular space and dissipation caused by the scraping of the blades. Furthermore, it was assumed that the heating of the liquid between the edge of the blade and the tube wall is adiabatic.

Leung (1967) gave a simplified correlation for determination of energy dissipation as:

$$Po^* = 15620 \, Re_R^{-1} B^{0.59} \tag{6}$$

without taking into account shaft diameter.

Penny and Bell (1967) attempted to incorporate existing results into a consistant picture, which in their opinion was superior to earlier correlation for design and scale up. Previous correlating attempts mostly involved fitting experimental data with dimensionless groups with little or no consideration given to how the final correlation represented physical reality. A dimensionless correlation should never be used without first reducing it to basic parameters so that one can judge if it predicts what one knows to be true. They revised the correlations for power requirements on a screw extruders and envisaged that the power number for agitators which are nearly as long as containing vessel should be based on D^4L rather than D^5 in the conventional power number. Correlating data with this new power number will result in power scale up as D^2 rather than D^3. They further mentioned that in case of variable clearance devices such as the votator, additional theoretical and especially experimental works are needed.

Trommelen and Beeck (1971) conducted studies to ascertain the mechanisms governing power consumption and to determine the influence of design and operating conditions on its magnitude. Power consumption was mesaured under various operating conditions of a scraped surface heat exchanger ($L = 0.46$ m. $D = 0.076$ m) by recording the torque and the rotational speed of the shaft. For most stirrers, the power number is a unique function of th rotational Reynold's number. However, this is not valid for scraped surface heat exchangers. For the same value of Re_R, Po increases with decreasing viscosity. This is caused by two phenomena: (1) Due to viscous dissipation the temperature of the liquid between the edge of the blade and the wall increases and therefore, the viscosity decreases. (2) The clearance between the edge of the scraper blade and the tube wall is dependent on the operating conditions and physical properties of the fluid.

A model incorporating these effects was derived for the power consumption:

$$Po = B\pi^2 \, Re_R^{-1} \sqrt{\frac{Y(nm'/\mu) + \beta}{1 + \mu n^{1.75}[Y\,(nm'/\mu + \beta)^{0.25}y]}} + f(Re_R) \tag{7}$$

Where $f(Re_R) = R \, Re_R^{-1}$ for $Re_R < Re_{Rcr}$. $\tag{8}$

$$f(Re_R) = R \, Re_{Rcr}^{-0.5} \cdot Re_R^{-0.5} \text{ for } Re_R > Re_{Rcr} \tag{9}$$

the parameters in the equation, Y, β, y and R are determined experimentally).

An empirical equation, more readily evaluated was derived in the form:

$$P' = \frac{251(nD)^{1.79} \mu^{0.66} B^{0.68} L}{(D - D_s)^{0.31}} \quad (10)$$

$$\text{(in M.K.S. Units)}$$

An attempt was also made to see whether power consumption measurements for another scraped surface heat exchanger (L = 0.26 m; D = 0.05 m) reported previously (Trommelen and Boerema 1966) could be correlated in the same way and it resulted in:

$$P' = \frac{276(nD)^{1.52} \mu^{0.68} B^{0.68} L}{(D - D_s)^{0.31}} \quad (11)$$

$$\text{(in M.K.S. Units)}$$

The Eq. 10 is only slightly different from Eq. 11.

Weisser (1972) developed a prediction equation accounting viscosity effects at wall temperature.

$$Po = 1066 \ Re_R^{-1.2} B^{0.47} (\mu/\mu_w)^{0.66} \quad (12)$$

$$\text{for } 150 < Re_R < 6100$$

$$\mu_w = \text{viscosity at wall temperature.}$$

HEAT TRANSFER

Kool (1958) and Hariott (1959) visualized heat transfer mechanism on the basis of penetration theory. It was assumed that portions of the liquid adjacent to the heat transfer surface are essentially stagnant, except when disturbed by the passage of scraper blade. Heat is transferred to the viscous liquid by unsteady state conduction. As the time between disturbances is short, the heat penetrates only a small distance into the stagnant liquid, and the process is exactly analogous to unsteady-state heat transfer to semi-infinite solid. The correlation based on this is as follows:

$$h_s = 2 \ \sqrt{\frac{K \ C_p n \rho B}{\pi}} \quad (13)$$

Equation (13) shows that the heat transfer coefficient depends on the thermal properties of the liquid and the agitator speed and implies that it does not depend on the viscosity of liquid or its velocity through the exchanger. Although Eq. (13) gives a good approximation in many cases, it is somewhat of an oversimplification. The liquid, especially if viscous, is not immediately deposited on heat transfer surface behind the scraper blade, as assumed in the development of above equation. The heat transfer coefficient is therefore somewhat affected by changes in liquid viscosity and the axial velocity. Skelland (1958) correlated experimental data during cooling of glycerol water and other glycerol oils in the form:

$$Nu = 4.9 \ Re_a{}^{0.57} \ Pr^{0.47} \ (Dn/\bar{v})^{0.17} \ (D/L)^{0.37} \qquad (14)$$

$$for \ 44 < Re_R < 2030, \ 705 < Pr < 2090, \ B = 2$$

Skelland *et al.* (1962) further investigated the effect of rotation, axial flow, mutator diameter, number of blades, and flow properties and obtained a correlation:

$$Nu = \alpha \ Re_a \ Pr^\beta \ (Dn/\bar{v})^{0.62} \ (D_s/D)^{0.55} \ B^{0.53} \qquad (15)$$

$$79 < Re_R < 494, \ 1000 < Pr < 4000$$

$$12600 < Re_R < 26200, \ 6 < Pr < 70$$

For cooling viscous liquids, $\alpha = 0.014$ and $\beta = 0.96$; for cooling thin mobile liquids, $\alpha = 0.039$ and $\beta = 0.70$. The functional dependence of the heat transfer coefficient on the parameters of the correlation is as follows:

$$h_s \approx \frac{(D - D_s)^{1.0} \ D_s{}^{0.55} \bar{V}^{0.38} \ n^{0.62} \ B^{0.53} \ K^{1-\beta} \ C_p{}^\beta}{D^{0.39} \ \mu^{1-\beta}} \qquad (16)$$

It is immediately apparent that the correlation can at most be applicable for the range of experimental data upon which it is based because h_s should not approach zero as D_s, $(D - D_s)$, and $\bar{V}$ approach zero. A more basic objection is that for viscous fluids the correlation predicts $h_s \approx K^{0.04}$, which is most unlikely. The axial flow velocities were in the range of 0.25 to 1.5 ft/min (1 to 4 min residence time), whereas the rotor speed varied from 100-750 rpm, giving a rotational Reynolds number about two orders of magnitude higher than axial flow Reynolds number. It is likely that

any potential effect of axial flow on the heat transfer coefficient would be completely overshadowed by the effect of rotation.

Latinen (1959) also developed a model based on penetration theory. For two-blade rotor, a correlation in dimensionless form obtained was:

$$Nu = 1.6 \ Re_R^{0.5} \ Pr^{0.5} \tag{17}$$

Latinen checked the penetration theory with the data of Houlton (1944) and Skelland (1958). The date of Houlton checked well, but those of Skelland did not. The data of Houlton were in the turbulent regime ($Re_a > 10,000$) and those of Skelland in transition regime ($10 > Re_a > 10,000$). Latinen concluded that the heat transfer mechanism in the transition regime must be different from that implied by assumptions of penetration theory. However, the agreement of data with penetration theory in the turbulent regime is to some extent fortuitous because the penetration theory model predicts that heat transfer is independent of viscosity. For scale up there seems to be general agreement that $h_s \approx n^{0.5}$.

Dinglinger (1964) generated data on cooling of Sucrose solution (temp. $-10\,°C$, $-15\,°C$) and developed a correlation in the form

$$Nu = 0.487 \ Re_R^{0.652} \ Pr^{0.33} \tag{18}$$

$$2000 < Re_R < 16000, \ 7 < Pr < 55 \quad B = 2$$

Skykora and Navratil (1966) performed experiments on heating of mineral oil using steam as heating medium. Following model for heat transfer coefficient was obtained:

$$Nu = 0.478 \ Re_a^{-0.01} \ Re_R^{0.48} \ Pr^{0.4} \ B^{0.24} \tag{19}$$

$$0.6 < Re_R < 132, \ 4300 < Pr < 21,700$$

Ghosal *et al.* (1967) correlated various process variables and physical properties of the liquids with the scraped surface heat transfer coefficient during heating. The liquids used were water, undiluted molasses, molasses-water solution and glycerol-water solution. The various variables were brought together in dimensionless groups and emperically correlated as:

$$Nu = 0.123 \ Re_a^{0.79} \ Pr^{0.6} \ (Dn/\bar{V})^{0.65}$$

$$5 < Pr < 2580 \quad B = 2 \tag{20}$$

Longitudinal Temperature Distribution

In arriving at most of the above correlations, it was assumed that overall heat transfer coefficient remained constant. However, the validity of the assumption could not be shown. The temperature distribution becomes linear when heat flux is uniform and logarithmic when one of the fluids is maintained at a constant temperature. Blaisdell *et al.* (1959) studied the effects of flow rates, rotor speed and temperature rise on the temperature distribution of water in a steam heat laboratory model scraped surface heat exchanger. When the temperature profiles were compared with a theoretical logarithmic rise, assuming the jacket heads to be but half effective, apparently rapid reduction in the canonical temperature, u, was observed. The logarithm of the cannonical temperature u, was then essentially linear with length.

However, this was not true at low flow rates, indicating that some assumption was violated. It was concluded that further work is required in order to determine the importance of the change in slope of semi-logarithmic heating curves observed at low flows and moderate to high tempreature rises. If the change in slope is caused by heat being conducted along the heat transfer surface and the rotor or due to condensate build up in the jacket, the effect would not be significant in a well drained commercial unit if the canonical temperature is large. Supplemental test data together with the literature suggested that a change in mixing effectiveness was probable cause. It was noted that axial conduction in exchanger walls can affect the longitudinal temperature distribution in the exchanger.

Effect of Axial Dispersion of Heat on Mean Temperature Difference (MTD)

According to Penny and Bell (1967), the previous investigators neglected the effect of axial dispersion of heat on the temperature distribution in agitated exchangers, except in pot type equipment where complete back mixing is generally assumed. All correlations have been based on the logarithmic mean temperature difference (LMTD), which assumes (1) no back mixing, (2) constant heat transfer coefficient, (3) negligible conduction within and along the channel walls and (4) no axial dispersion. The quantitative effect of axial dispersion on MTD cannot be predicted. In general, as axial flow rate decreases and the rotor speed increases, relative axial dispersion increases and the true MTD decreases. No work has been directed towards mixing on MTD in agitated exchangers. However, considerable work has been done to characterize axial mixing as a function of system parameters. Most of the work has related to predicting the ef-

fect of axial mixing on diffusion and chemical reaction. On the basis of
the analogy between transfer of heat and mass, one would expect that the
dispersion mode (analogous to Fick's law of diffusion) would also suffice
for heat transfer. White and Churchill (1959) have given the differential
energy equation for a stream with heat transfer by axial dispersion model;

$$K_E \, \frac{d^2T}{dZ^2} - \frac{W}{A} \, \frac{dT}{dZ} = \frac{h_s P}{A} \, (T - Tw) \tag{21}$$

Smoot and Babb (1958) solved the mass transfer analogy of this equation
for diffusion between parallel flowing streams.

Some qualitative facts regarding axial dispersion are (1) for constant
heat transfer coefficients, the MTD is always intermediate between that
for complete mixing and for no axial dispersion, (2) axial dispersion in
a stream will have no effect on the MTD if the stream is isothermal in
exchanger. Axial dispersion has greatest effect on the MTD as the ratio
of inlet ΔT to outlet ΔT departs increasingly from unity, (3) axial disper-
sion as a result of back-mixing has an increasing effect on MTD as axial
flow rate decreases and rotor speed increases, (4) as a decreased axial flow
rate increases the effect of backmixing, thereby lowering the MTD, the
data using LTD, often show a false dependence of the heat transfer coef-
ficient on axial flow rate, (5) the results obtained by Skelland et al. (1962)
are valid only for high axial flow rate at any particular rotor speed because
back mixing affects the MTD least for high flow rates.

Bott et al. (1968) studied the effects of axial dispersion on heat transfer.
A theoretical axial diffusion model was presented which predict the effect
of back mixing on overall rate of heat transfer in terms of two dimen-
sionless quantities N_T and Pe (30 to 100), and where $N_T = h_s AT/W C_p$.

The axial dispersion coefficient was determined experimentally and the
measured values were empirically correlated. Within the defined range
of operating conditions, back mixing has negligible effect on heat transfer
rates. They suggested that the design of scraped film processor may be
based on the assumption of plug flow. For a heat exchanger correspond-
ing to a value of $N_T = 2$, the effect of back mixing would be to reduce
the rate of heat transfer predicted on the assumption of plug flow by almost
5%. The effect of back mixing in a full tube exchanger would be much
larger than that in the scraped film equipment because the main axial vel-
ocity is normally lower i.e., liquid residence time is longer. However,
it is argued that the assumption of plug flow may lead to too high values
for driving force for heat transfer. Axial dispersion decreases the effec-
tive temperature difference between working fluid and heat transfer

medium. For the plug flow with axial dispersion model the decrease in heat transfer rate is dependent on two dimensionless parameters N_T and Pe. The mass flow rate has a great influence on the decrease in heat transfer due to axial dispersion. A decrease in flow rate results in increasing N_T and decrease in Pe ($\bar{V}$ is proportional to W).

Cuevas (1981) had reported that neglecting the effect of axial dispersion could result in an error in overall heat transfer coefficient at lower mass flow rates. Such values of overall heat transfer coefficient have been referred as 'apparent.'

Maingonnat and Corrieu (1983) studied the effect of axial heat diffusion on thermal performance. It was reported that the diffusion, as shown by measurement of product temperature, could be characterized by sudden increase of the product temperature just after the entrance in the exchanger. This phenomenon was more pronounced at high rotor speeds, increasing number of blades and decreasing flow rates. The rate of increase of the product temperature at the exchanger inlet was proportional to the ratio of rotor speed to the mean axial velocity of the product. This ratio could be used as a correcting factor for an error made when the mean difference in temperatures between fluids is estimated by means of logarithmic mean temperature difference (LMTD). In other words, this error has repercussions in the computation of heat transfer coefficient.

Trommelen (1967) investigated the mechanism of heat transfer based on penetration theory and added a correction factor; f to Latinen's expression assuming complete mixing.

$$Nu = \frac{128 \ f \ D^2 \ n \ B \ C_p}{K} \qquad (22)$$

$$\text{where } f = 1 - 3.28 \ Pe^{-0.22} \ (Pe < 1500)$$

$$80 < Re_R < 8000, \ 119 < Pr < 2650 \qquad (23)$$

For Pe > 2500, f was function of Pr. However, from experimental data of Skelland et al. (1962) it followed that

$$f = 1 - 2.78 \ (Pe + 200)^{-0.18} \qquad (24)$$

Equations (23) and (24) are very similar.

Sykora et al. (1968) studied heat transfer in laminar and transitional regimes and evolved the expressions;

$$Nu = 0.8 \ Re_R^{0.36} \ Pr^{0.37} \ B^{0.25}$$

$$1.15 < \text{Re}_R < 44, \quad 55 < \text{Pr} < 200,000 \tag{25}$$

and

$$\text{Nu} = 2.0 \; \text{Re}_R{}^{0.48} \; \text{Pr}^{0.24} \; B^{0.15}$$

$$44 < \text{Re}_R < 7940. \tag{26}$$

Penny and Bell (1969) experimented in a thermal processor agitated with a fixed clearance thin flat blades. Ethylene glycol and mineral oil were used as test fluids and heating was done through electrical energy. It was observed that the clearance between blade tip and vessel wall had practically no effect on heat transfer in turbulent regime. In this regime, the wall temperature profile was almost linear except for skewing down at the exchanger ends. In laminar regime, the wall temperature profile skewed down at both ends and more so near the entrance. The experimental data were correlated in the form;

$$\text{Nu} = 0.123 \; \text{Re}_R{}^{0.78} \; \text{Pr}^{0.33} \; (\mu/\mu_w)^{0.18}$$

$$\text{Re}_R > 400, \quad B = 2 \tag{27}$$

Trommelen et al. (1971) further investigated the heat transfer mechanism during cooling of water and glycerol solutions and concluded as follows:

(1) The heat transfer involves three steps: (1) Penetration of heat by conduction in a thin-layer close to the heat transfer surface in the time between two scrapings. (2) Partial temperature equalization in the boundary layer that builds up on the blade. (3) A convective radial transport from the heated layer to the bulk of the liquid Taylor vortices contribute to this radial transport.

(2) The heat transfer coefficient can be described by an equation resulting from penetration theory and modified by an empirically determined correction factor:

$$\text{Nu} = 1.33 \; (\text{Re}_R \; \text{Pr} \; B)^{0.5}f \tag{28}$$

The correction factor 'f' includes (1) the incompleteness of temperature equalization in the boundary layer, (2) the effect of radial dispersion and (3) the decrease in driving force for heat transfer due to axial dispersion. 'f' has been found to be a function of Prandtl number when the influence of axial dispersion of heat transfer is absent (high mass flow rates).

$$f = 2.0 \ Pr^{-0.25} \text{ for } Re_R > Re_{Rcr} \tag{29}$$

$$350 < Pr < 4200$$

Below Re_{Rcr}, 'f' decreases due to insufficient radial mixing.

At low mass flow rates the influence of the axial dispersion on the driving force for heat transfer is important. The apparent decrease in the heat transfer coefficient due to this effect can be calculated. To do so the plug flow with dispersion model is applied, in which the axial dispersion coefficient is calculated from the standard deviation of the residence time distribution curve.

(4) High heat transfer coefficients are obtained when Taylor vortices are present ('f' becomes independent of Re_R). A wide annular space prompts instability. The power dissipation in the annular space is less when the value of $D - D_s/D$ is high. For the same value of blade velocity the flow is more unstable in a heat exchanger with small tube diameter than in one with a large tube diameter. If a certain heat transfer area is required, a long narrow heat exchanger is preferable to a short large diameter apparatus. The increase in pressure drop can be reduced by increasing the annular space.

(5) If a heat exchanger is operating in Taylor vortices regime, the heat transfer coefficient is proportional to the square-root of the number of scrapings for unit time. However, the power consumption in annular space is proportional to $n^{2.5}$ and independent of number of rows of scraper blades. The heat removal is therefore more efficient with a large number of blades and a low shaft speed than with a smaller number of blades and a higher shaft speed, given the same number of scrapings per unit of time in both cases. The same is true in the Couette regime. Theoretically, then, the best way of scaling up is by increasing the length of a scraped surface heat exchanger. Increase in pressure drop can be offset by widening the annular space. The heat transfer coefficient does not change and the power consumption increases linearly with length. When for an increase in mass flow rate the length of votator is increased proportionally, the power consumption per unit mass of working fluid retains its value.

(6) if the heat transfer area is increased by increasing the diameter of the heat exchanger, the shaft speed should be reduced. If the blade velocity and the annular space are the same for the small and large unit, the power consumption is higher for large unit. By increasing the number of rows of scraper blades the heat transfer coefficient can be kept constant. A check should be made to ascertain whether the flow regime is same for both the cases.

Weisser (1972) conducted experiments on cooling of water and sucrose solutions using methanol as cooling medium. The data were processed in the form:

$$Nu = 1.2 \ Re_R{}^{0.5} \ Pr^{0.33} \ B^{0.25} \tag{30}$$

$$150 < Re_R < 18,000 \text{ and } 7 < Pr < 200.$$

Koziol and Jaros-law (1978) and Peter *et al.* (1979) also developed Nusselt type heat transfer correlations. Former have modified the equation by introducing a correction factor determined from the published data on heat transfer. Ramdas *et al.* (1980) investigated the heat transfer coefficient for a commercial size (6 in. ID and 6 ft. length) S.S.H.E. Three test fluids, two petroleum lube fractions and corn syrup, were used. The flow rates were varied from 0.8 to 5.8 gpm and rotor speed from 0.5 to 25 rpm.

The experimental data for heating and cooling were correlated in the form:

$$Nu = 57 \ Re_a{}^{0.059} \ Re_R{}^{0.113} \ Pr^{0.063} \ (\mu/\mu_w)^{-0.018} \tag{31}$$

in the range $0.0016 < Re_a < 0.23$ and $0.0164 < Re_R < 68.65$

The experimental values of film coefficient were compared with those predicted by the penetration theory model and it was indicated that this model is unsuitable for predicting the scraped film coefficients for viscous fluids in laminar flow.

Cuevas *et al.* (1982) studied the performance characteristics of a pilot scale SSHE (Contherm Model 6x2) under ultra high temperature (UHT) conditions. Outlet temperature and overall heat transfer coefficient were significantly affected by rotational speed of blades, mass flow rate, inlet temperature, steam pressure and solids content of model low acid food (soy extract). The scraping action of blades greatly improved the performance upto about 350 rpm; higher blade speeds had little or no effect and sometimes decreased U_h values. Application of dimensionless models showed that the effect of axial Reynolds number (Re_a) on Nusselt number was more significant in the turbulent regime ($Re_a > 1800$). While rotational Reynolds number (Re_R) above 10^5 had a significant effect at all values of Re_a between 1200 and 3700. The following correlations were obtained for water:

$$Nu = 0.304 \ Re_a{}^{0.504} \ Re_R{}^{0.322} \ Pr^{0.33} \ (\mu/\mu_w)^{0.18} \tag{32}$$

$$Re_a < 1800; \ 8600 < Re_R < 3.7 \times 10^5; \ 1.76 < Pr < 2.9$$

$$Nu = 4.599 \times 10^{-4} Re_a{}^{0.942} \ Re_R{}^{0.637} \ Pr^{0.33} \ (\mu/\mu_w)^{0.18} \tag{33}$$

$$Re_a > 1800; \ 35000 < Re_R < 3.81 \times 10^5; \ 3.05 < Pr < 5.16$$

for Soybean-water extracts of solid contents varying from 1.65 to 6.14.

$$Nu = 0.0982 \ Re_a^{0.468} \ Re_R^{0.4} \ Pr^{0.33} \ (\mu/\mu_w)^{0.18} \tag{34}$$

$$Re_a < 1800.$$

At very high values of the Taylor number ($> 10^{10}$) when laminar axial flow conditions prevailed, Nusselt number decreased with increase in rotational Reynolds number.

Heat transfer characteristics in the above SSHE were further investigated by Cuevas and Cheryan (1982) with reference to the application of penetration theory and Wilson plot models. The penetration theory of Harriot (1958) predicted 'h_s' values quite well at low axial mass flow rates where laminar flow conditions prevail. However, turbulent axial flow resulted in experimental 'h_s' greater than predicted by the theory. A correction factor based on Prandtl number suggested by Trommelen et al. (1971) did not improve the prediction. The Wilson plot method was useful for explicitly accounting axial and rotational velocity effects, both of which significantly affected heat transfer, especially at high values of either variable. The models derived from this analysis were of the form;

$$h_s = 1709 \ V_z^{0.42} \ V_e^{0.43} \ D_o/D_i \ *(for \ water) \tag{35}$$

$$h_s = 905.5 \ V_z^{0.22} \ V_e^{0.33} \ S^{-0.16} \ D_o/D_i \tag{36}$$

V_z and V_e are axial and rotational components of velocity field. Boxtel and Geothart (1984) conducted experiments in a water cooled scraped surface heat exchanger to study the heat transfer to water and some highly viscous starch based foods viz. veloute sauce, roux and regout. The heat transfer coefficient on jacket side (h_j) was computed from a modified overall resistance equation i.e.,

$$\frac{1}{U_h} = \frac{1}{cn^{0.5}} + \frac{x}{K_w} + \frac{1}{h_j} \tag{37}$$

(c is a constant).

'h_s' in the above equation was substituted by $cn^{0.5}$ because if the shaft speed is varied and the other conditions remain unchanged then hs varies directly as $n^{0.5}$ as followed from penetration theory model suggested by Trommelen et al. (1971). The experimental values of 'h_j' fitted in the correlation;

$$Nu = 0.0158 \ Re_a{}^{0.8} \ Pr^{0.8} + 18.2 \tag{38}$$

(For jacket side)

The values were close to the one predicted by the conventional Dittus-Boelter equation. A maximum deviation of about 10% appeared at the lower and higher Reynolds numbers. For water and three starch based food products (starch content 12-18%) the heat transfer coefficients inside the scraped cylinder were measured for shaft speeds ranging from 1.67 to 10 rps. The experimental results were compared with heat transfer coefficients calculated with a model based on penetration theory (Trommelen *et al.* 1971). The product mass flow rate did not affect heat transfer coefficients. In the shaft speed range studied the heat transfer coefficient at scraped surface varied from 3200 to 7800 W/m^2K for water, from 500 to 3150 W/m^2K for veloute sauce, from 670 to 1330 W/m^2K for roux and from 780 to 1900 W/m^2K for ragout. This study has supplied useful data on heat transfer to highly viscous non-Newtonian food products in which very little information is available.

CONCLUSIONS

On the basis of review, following inferences are drawn; (1) The flow pattern in liquid full SSHE is either in the couttee flow or in the form of Taylor vortices depending upon laminar or turbulent regime. The stream lines are concentric circles outside the blade area and they contract and expand near the blades. The resulting velocity profile is similar to that between two parallel plates, one of which is moving with a pressure gradient in the direction of flow. The low axial flow rates ($Re_a < 40$) and high rotational speed result in maximum back mixing effects. However, these effects minimize at high axial flow rates and the residence time distribution function approximates more closely to plug flow. (2) The data in the turbulent regime have been shown to agree with penetration theory. This agreement may be, to some extent accidental because the penetration theory predicts that heat transfer is independent of viscosity, where as the heat transfer significantly depends on viscosity in the turbulent regime also. The penetration theory generally gives high coefficients in transition regime. (3) The effect of axial dispersion of heat is to lower the mean temperature difference (MTD) and its effect due to back mixing (axial convection) is greater for low axial flow rates and high rotor speeds. (4) High heat transfer coefficients are obtained when Taylor vortices are present. (5) For scale up, there seems to be general agreement that h_s varies directly as $n^{0.5}$. (6) In using the correlation of Skelland, one should do so for the highest

axial flow rates for any particular rotor speed because back mixing affects the MTD least for highest flow rates. (7) The models for power consumption and heat transfer which account for viscous dissipation, clearance and the effect of axial dispersion would be more appropriate to use. (8) As there is a wide variation in the values of the exponents of dimensionsless numbers essentially because these correlations were established for defined exchanger designs and experimental conditions, extrapolation needs to be done with great caution. (9) In a floating blade type rotor, the clearance between the edge of scraper blade and the tube wall varies with the operating conditions (including blade mass and geometry) and the physical properties of the fluid. Its effect on h_s needs further investigations.

NOMENCLATURE

A_t	=	area for heat transfer.
A	=	Area of flow channel.
B	=	Number of blades (rows).
C_p	=	specific heat of liquid.
D	=	diameter of rotor/inside diameter of shell.
D_s	=	diameter of shaft.
h_s	=	scraped film heat transfer coefficient.
h_j	=	heat transfer coefficient on jacket side.
K	=	thermal conductivity of liquid.
K_w	=	thermal conductivity of wall material.
K_E	=	effective thermal conductivity in the dispersion model.
L	=	length of heat exchanger/agitator.
n	=	rotor speed (r.p.s)
P'	=	power consumed by rotating shaft and blades.
P	=	Perimeter of flow channel.
T	=	Temperature of heating medium.
T_w	=	Wall temperature.
U_h	=	overall heat transfer coefficient.
$\bar{V}$	=	average fluid velocity.
W	=	axial mass flow rate.
x	=	wall thickness of heat exchanger.
Z	=	axial dimension.

Dimensional Groups:

Nu = Nusselt number $\left(\dfrac{h_s\, D}{K}\right)$

Pe = Peclet number $\left(\dfrac{\bar{V}(D - D_s)\, C_p}{K}\right)$

Po = power number $(P'/\rho n^3\, D^4 L)$ or $(P'/\rho n^3\, D^5)*$

Pr = prandtl number $(\mu C_p/K)$

Re_a = axial Reynolds number $\left[\dfrac{\rho(D - D_s)\, \bar{V}}{\mu}\right]$

Re_R = Rotational Reynolds number $(D^2 n \rho/\mu)$.

Re_{Rcr} = critical rotational Reynolds number.

Ta = Taylor number $\left[\dfrac{(n^2\, \rho^2\, (D - D_s)\, D_s{}^2)}{\mu^2(D + D_s)}\right]$

Ta_{cr} = Critical Taylor number.

Greek Letters:

μ = viscosity of liquid.

ρ = density of liquid.

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HYDRODYNAMICS AND HEAT TRANSFER IN THIN FILM SCRAPED SURFACE HEAT EXCHANGERS — A REVIEW

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ABSTRACT

The available literature on hydrodynamics and heat transfer in thin film scraped surface heat exchangers was reviewed and arranged in concise form. The hydrodynamics covers fluid flow, residence time distribution, power consumption and film thickness. In heat transfer, sensible heating and evaporation aspects are explored. The constraints of various mathematical models are explained. The problems needing further investigation are also delineated.

INTRODUCTION

Thin film scraped surface heat exchangers (SSHE) have been in use in food industry for many years because of their unique characteristics of handling difficult process fluids. The problems like viscosity variations, heat sensitivity and foaming, encountered during processing of food pro-

ducts, can be easily handled in thin film SSHEs. Figure 1 and Fig. 2 show few configurations of thin film SSHEs and rotor designs, respectively. Fischer (1965), Hauschild (1969), Smith (1972), Holdt (1978), Freeze and Glover (1979), Angel and Baird (1983) and Arlidge (1983) had detailed the process applications and performance characteristics of thin film SSHEs. To understand why the thin film SSHEs successfully process such fluids, one must consider the basic operations that occur in it viz. film hydrodynamics, heat and mass transfer. The information on hydrodynamics and heat transfer aspects is widely scattered. The objective of this paper is to present the literature in concise form.

HYDRODYNAMICS

Flow Pattern

The agitation produces a complex liquid flow pattern when compared to "Standard" heat exchangers. The usual flow resulting from gravity or some pumping mechanism is greatly complicated by the action of agitator blades flowing through the liquid film. Harper (1960) reported results of an investigation of the performance of a vertical thin wiped film evaporator in concentrating apricot, peach and pear purees and a study on the viscosity characteristics of the products. It was found that the viscosity behavior was well represented by the equation:

$$\tau = m \ (\gamma)^n \tag{1}$$

where, m = consistency coefficient,
      n = power index.
      τ = shear stress,
      γ = shear rate.

The value of m increased in a regular manner with concentration, while variation in n was small. Overall heat transfer coefficients correlated well with values of m, and performance was found much better for non-Newtonian fluids than for Newtonians of equivalent consistency. This could be due to the fact that the viscosity of a Newtonian would have to be very high to give the same consistency as the non-Newtonian.

Mutzenberg (1965) studied fluid flow in a vertical unit in which rotating shaft was provided with four nonscraping blades. The rotor speed was 10 to 15 m/s. The blades pushes a "bow wave" having a zone length determined by product properties and the vertical liquid flow rate. Zone lengths
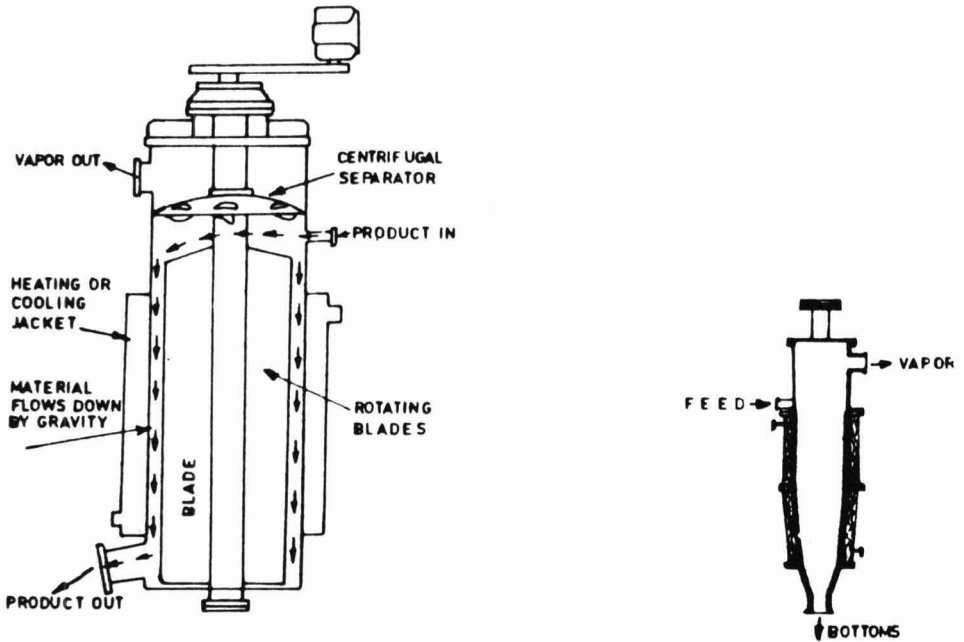
VERTICAL THIN-FILM HEAT EXCHANGERS
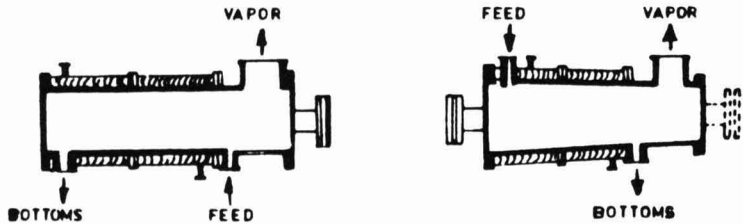
HORIZONTAL THIN-FILM HEAT EXCHANGERS

FIG. 1. MECHANICALLY FORMED THIN FILM SSHEs
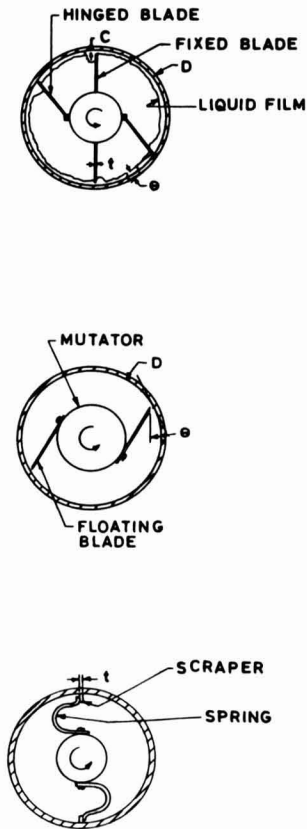
FIG. 2. S.S.H.E. ROTORS

of 2.5×10^{-2} m to 10×10^{-2} m were observed. Adjacent to bow wave is highly turbulent "Squeeze zone" followed by a "tranquilizing" zone; this three-stage cycle is repeated by the next blade attached to the vertical rotor.

Godau (1974) investigated theoretically the hydrodynamic processes in the wiped-film evaporation of polymer solutions. The analysis proceeded from the known fact that the film is thicker than the gap between the rotor and the stator. The gravitational and tangential flows, the bow wave, the couette flow in the gap, the trailing vortex and the calming zone were considered. The approximate and exact solutions could be obtained for the film thickness by Navier-Stokes differential equations.

Residence Time Distribution

The integral liquid-flow behavior was studied using tracer technique by Danckwerts (1953) and Levenspiel (1962). A small amount of electrolytic

solution was injected into a continuous liquid feed stream to a thin film unit. This unit had a conductivity cell located at the outlet. The continuously recorded conductivity values give the distribution of the initially injected solutions. This residence time distribution is an effective way to characterize a continuous processing unit since the curve will vary depending on the liquid involved. The typical results were then transformed into dimensionless quantities: Residence time divided by mean residence time $(t/\bar{t})$; conductivity values divided by mean conductivity $(C/\bar{C})$ (when all injected solution would be evenly mixed in a liquid amount equivalent to total liquid hold up). Then $C/\bar{C}$ versus $t/\bar{t}$ curves were plotted. Following inferences were drawn:

Compared to an agitated thin film, the falling film has far less internal turbulence, and it is this turbulence that favors heat and mass transfer.

If there was short circuiting in the agitated thin film evaporator (vertical flow rate in the bow wave much faster than vertical flow in thin film on the wall) the time distribution pattern would differ from the actual one. This leads to the conclusion that the bow wave is constantly renewed and does not consist of a liquid bar superimposed over liquid film on the wall of the evaporator. The experimental studies in transparent model of agitated thin film units confirm this conclusion.

Parker (1965) had explained the factors controlling residence time for specific applications. Apart from major factors viz. L/D ratios and the rotor speed, feed rate can provide some degree of residence time control. Also by placing a 'dam' like overflow in form of 'retention' ring at concentrate or residue end of the thermal section, complete film control could be restored. The film build up could be varied by width of the ring to avoid surface starving.

Freeze and Glover (1979) have cited a vertical design which include a 'residence time control' ring at the end of thermal surface to hold back (and thus build up) the film thickness. Following graphical presentations for such exchangers have been shown: (1) Mean residence time $(\bar{t})$ at various rotor speeds versus dimensionless ratio of residence time ring width to standard residence time ring width (b/b_o). (2) Relative frequency of fluid elements per unit time $(\underline{R})$ versus specific residence time $\underline{t}$.

The feed rate in the evaporator was 0.213 kg/s m².

It was observed that as b/b_o increases, $\bar{t}$ increases. For same value of b/b_o, mean residence time increases with rotor speed.

Lin (1979) has discussed the theory of residence time distribution of flow and its application to the estimation of the sterilization efficiency of process equipment. He concluded that for flows in complex sterilizer units like plate heat exchangers and scraped surface heat exchangers which have either complex channel geometries or moving accessory along the flow passage, experimental determination of the residence time distribution

is necessary because the flow patterns for these instances become too complicated to be adequately described by a mathematical model.

Power Requirement

In thin film equipment, the clearance between the agitator and vessel wall is either fixed or variable because both rigid and hinged agitators are used. In variable clearance exchanger, the agitator blades are forced towards the vessel wall by springs or centrifugal action. The "Slipper bearing effect" acts to hold the agitator off the wall. The clearance varies with operating conditions.

Energy induced by the rotating blade is mainly consumed by internal friction or turbulence within the liquid.

Kern and Karakas (1959) have used slipper bearing theory to compute the clearance power (P_c) requirements of a centrifugally loaded beveled-edge agitator. They arrived at the following expression for power which is supposed to hold as θ approaches zero.

$$P_c = 16\pi^3 \ M \ D^2 \ N^3 \ \tan\theta \ \ldots \ldots \ldots \ \text{ft-lbf/s} \qquad (2)$$

$$\theta = \text{angle between agitator and tangent to the vessel.}$$

This expression does not account the lift force due to the pressure increase under the bearing. The following expression was rigorously derived from slipper bearing equations by Norton (1942) and Streeter (1958)

$$P_c = \pi^2 \ D^2 \ N^2 \ \mu L \ \left[\frac{t}{c}\right] \ \left[\frac{1}{a-1}\right] \ \left[4 \ \ln a - \frac{6(a-1)}{a+1}\right]$$

$$\text{ft-lb}_f/\text{s} \qquad (3)$$

Equating the lift force on the bearing to the centrifugal force relates, M, c and a

$$M = \frac{3L\mu}{N} \ \left[\frac{t}{c}\right]^2 \ \left[\frac{1}{a-1}\right]^2 \ \left[\ln a - \frac{2(a-1)}{a+1}\right] \qquad (4)$$

where, t = circumferential thickness of agitator near the vessel wall, ft.

M = Mass of agitator, lbm.

$$a = \frac{c_m}{c}$$

c = Minimum clearance between agitator and vessel, ft.

c_m = Maximum clearance between agitator and vessel, ft.

D = Diameter of vessel, ft.

L = Length of agitator, ft.

Bhattacharya (1970) in a study on hydrodynamics of film formation, developed an expression for the hydrodynamic force (F_h) in scraped film. Hydrodynamic force is a function of film thickness and the centrifugal force is a function of blade mass and speed. By equating these two forces, for known values of film thickness, the required rotor speed can be determined for a definite system. Refer Fig. 3.

$$F_h = \frac{6\mu U_B}{(Sin\psi)^2} \left[ln(1 + C) - \frac{2C}{2 + C} \right] l; \text{ dynes.} \tag{5}$$

where $C = \dfrac{t \, sin\psi}{\delta Cos(\alpha + w)}$

The equation for centrifugal force (F_c) at the blade tip is:

$$F_c \, Sin\psi = \frac{1}{2}(M \, w^2 h \, Sin\theta) \, Sin\psi; \text{ dynes} \tag{6}$$

At equilibrium $F_h = F_c \, Sin\psi$

By substituting $w = U_B/R$, expression for U_B can be derived as follows:

$$U_B = \frac{12 \, R^2 \, l \, \mu}{M \, h \, Sin\beta \, (Sin\psi)^3} \left[ln(1 + C) - \frac{2C}{2 + C} \right] \text{ cm/s} \tag{7}$$

Where:

U_B = Linear velocity of the blade tip, cm/s

t = Thickness of blade, cm.

w = Rotational speed of blades, radians/s

l = length of blade, cm.

L = Total length of evaporating surface, cm.
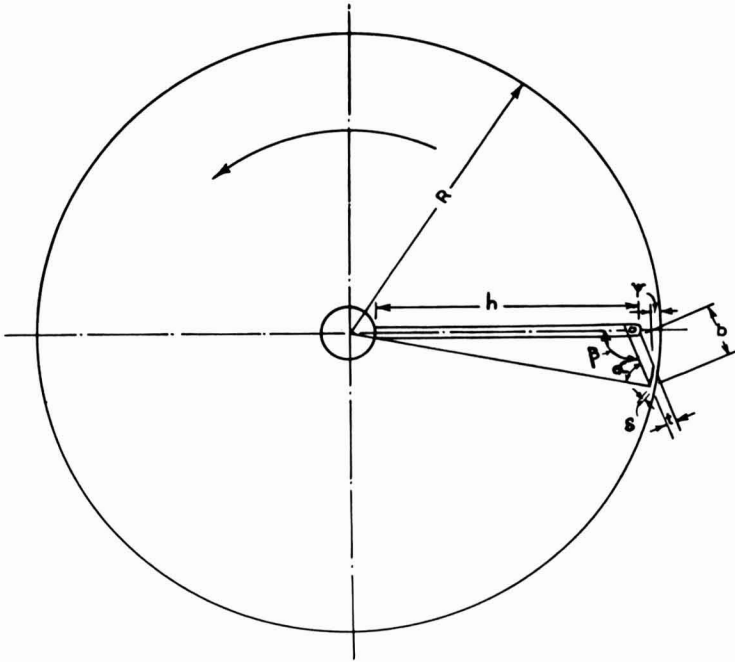
FIG. 3. HYDRODYNAMIC POSITION OF BLADE

R = Inner radius of heat exchanger, cm.

M = Mass of a blade g

The viscous drag (F_t) at the blade tip can be estimated from the following equation.

$$F_t = \frac{2U_B\mu}{Sin\psi} \left[2\ 1n(1 + C) - \frac{3C}{2 + C} \right] 1;\ dynes \qquad (8)$$

Power required to overcome the drag is p_c'

$$p_c = B\ F_t\ L\ U_B;\ dynes - cm/s \qquad (9)$$

Additional power (p_a) will be required to accelerate the fluid. It is assumed that a fraction S of the hold up is accelerated from rest to the velocity U_B once each revolution.

$$p_a = \frac{S\ H\ U_B^3\rho}{4\pi R}\ ;\ \text{dynes} \cdot \text{cm/s} \tag{10}$$

Holdup volume (H) consists of two volumes, namely holdup in the film and that in the fillet. H can be calculated from

$$H = 2\pi RL\delta + 3.8\sqrt{B}\left[\frac{V_f\mu}{g\rho} - \frac{2\ R\delta^2\pi}{3}\right]\Big/\ L;\ \text{cm}^3 \tag{11}$$

where V_f = volumetric feed rate cm³/s.
δ = Film thickness cm.

Average residence time t can be estimated as:

$$t = \frac{H}{V_f};\ \text{s.} \tag{12}$$

Mutzenburg (1965) cited that in commercial equipment, the power consumed by internal friction is approximately 1615-3230 W/m². This is equivalent to the pressure drop of a liquid flowing at 4 to 8 m/s through 2.5×10^{-2} m I.D. tube with a surface of 9.3×10^{-2} m². These comparable velocities are considerably higher than actual velocities in tube heat exchangers and explain the high heat transfer values in agitated thin film evaporators.

Film Thickness

Large number of correlations for the thickness of a nonboiling, undisturbed film are given in the literature. The thickness of a laminar flowing film can be determined from Nusselt's correlation:

$$\pi_\delta = 0.909(Re_f)^{1/3} \tag{13}$$

Similar correlations for thickness of turbulent film have been proposed. Brauer's correlation (1956).

$$\pi_\delta = 0.911\ (Re_f)^{8/15} \tag{14}$$

and Zivajkin's correlation (1961)

$$\pi_\delta = 0.141 \ (Re_f)^{7/12} \tag{15}$$

In a boiling mechanically affected flowing film distinctively different flow conditions compared to nonboiling flowing film exist. The character of film under boiling conditions depends to a great extent on the value of the specific heat flux. The vapor bubbles result in an intensive "apparent turbulence" in the boiling film with a simultaneous amount of film under going evaporation. In a film type rotary evaporator, with the usual vertical arrangement, a counter current flow of the evaporated vapors and the liquid film takes place. The counter-current flow causes a friction effect at the interface. This friction also effects flow conditions. Feind (1960) developed a correlation for calculation of the thickness of a turbulent flowing film under such conditions:

$$\pi_\delta = 1.068 \ (Re_f)^{\frac{1}{2}} \tag{16}$$

which can be used for $Re_f > 1600$ (critical Reynolds number = 1600).

Bhattacharya (1970) had used the following expression for calculating film thickness in a 'Sambay' type evaporator provided with swinging blades. The film thickness was assumed to be uniform throughout the heat exchanger and the heat transfer was principally due to steady state conduction.

$$\delta = \frac{K}{h_s} \tag{17}$$

Ujhidy and Babos (1967) started from the assumption that a flowing mechanically affected film consists of a laminar sublayer within a turbulent film. The velocity of liquid is a result of combined gravitational and circumferential forces and of the braking effect of the forces of internal friction. On the basis of experiments with distilled water in a film type rotary evaporator with 0.12 m² of surface provided with rotor with scraping blades, they developed the following correlation for turbulent flowing film:

$$\pi_\delta = 2.46 \times 10^{-2} \ (Re_f)^{\frac{1}{2}} \left\{ \frac{V_c^3 \rho}{g\mu} \right\}^{1/6} \tag{18}$$

which can be used under the assumption that

$$\mathrm{Re_f} \cdot \frac{V_c{}^3 \rho}{g\mu} > 10^8$$

However, Dieter and Hubner (1970), on the basis of their own experience with Sambay-type evaporator, criticized the above correlation that the circumferential velocity of the rotor blades has a controlling effect on the thickness of the film.

Zacharov *et al.* (1970) conducted experiments on a dimensionally atypical functional surface of a film type rotary evaporator (dia. 0.1 m, length 0.14 m) and they found by visual observation of the liquid through a transparent wall of the apparatus that the thickness of the film at every location of the surface pulsates with a frequency corresponding to the speed of the rotor. The results of experiments were used to develop the following correlations for film thickness.

$$\pi_\delta = 0.466 \ (\mathrm{Re_f})^{\frac{1}{2}} \ e^{0.1} \ \ln V_c \tag{19}$$

This correlation does not have as strong an effect of the speed of rotor as Eq. 18.

Domanskii *et al.* (1971) investigated flow conditions in a film-type rotory evaporator provided with a rotor with fixed blades. They assumed that the space between the edge of the blade and heat transfer area was filled with liquid and that the excess liquid is rotating in a frontal wave ahead of the moving blade. Because of the intense circulation in the frontal wave they assumed that the flow was turbulent. On the basis of experiments with distilled cold water, the following correlation was derived:

$$\frac{\delta}{D} = 0.75 \ \frac{C}{D} + 0.35(B)^{0.29} \left\{ \frac{M_o}{w^3 \rho D^4 L} \right\}^{0.14} \tag{20}$$

$$\left\{ \frac{F - F_z}{\mu} \right\}^{0.57} \left\{ \frac{w^2 D}{g} \right\}^{0.57} \left\{ \frac{w D^2 \rho}{\mu} \right\}^{-0.72}$$

where,

    C = Clearance between blade edge and inside surface, m.

    Mo = Motor output, watts.

    w = Angular velocity, rad/sed.

    F = Amount of liquid related to a unit length of circumference of the heat transfer area, kg/s.m.

    F_z = Amount of product related to a unit length of circumference of the heat transfer area, kg/s.m.

Reinemann et al. (1973) assumed that the film of liquid does not fill the space between the edge of the blade and the heat transfer area, and that it is filled by a laminar layer with connected turbulent layers. For the film thickness they derived the equation:

$$\delta = \left[\frac{0.023\mu D}{\rho N.(D - C)}\right]^{\frac{1}{2}} + \left[\frac{F}{0.304\rho.g} \left\{\frac{\mu N(D - C)}{\rho D}\right\}^{\frac{1}{2}} - \right.$$
$$\left. \frac{0.0077\mu D}{\rho N(D - C)}\right]^{\frac{1}{2}} \tag{21}$$

According to Frank and Lutcha (1980), a common shortcoming of the above described equations is the neglect of boiling and of the presence of the vapor phase which serves as the medium between the moving edge of the blade and the surface of the film. They prepared general correlations for film thickness, using experimentally determined mean residence time, to determine the variable thickness of a boiling, mechanically affected film. The experimental work was carried out in a film type rotary evaporator with a heat transfer area of 1 m². It had fixed blades with a clearance of 0.002 m between the edge of the blades and the heat transfer area. Liquids used were distilled water and sugar solutions. For conditions under which a laminar film flow can be assumed, the following correlation was determined at zero specific heat flow (nonboiling film):

$$\pi_\delta = 0.95 \ (Re_f)^{0.496} \ (Fr)^{-0.062} \tag{22}$$

which applies for $23.7 < Re_f < 1360.8$
and $20.1 < Fr < 1323.6$

For conditions under which turbulent flow of the film can be expected, the following correlation was derived:

$$\pi_\delta = 2.08 \ (Re_f)^{0.286} \ (Fr)^{-0.12} \tag{23}$$

which applies for $997.2 < Re_f < 2173.1$
and $336.2 < Fr < 2365.6$

For boiling turbulent film:

$$\pi_\delta = 0.368 \ (Re_v)^{0.368} \ (Fv)^{0.0052} \ (Wav/u)^{0.094} \tag{24}$$

Which applies for $4.7 < Re_v < 1753.1$
$$355.7 < F_v < 5916.10^{11}$$
$$227.4 < Wav/u < 8320.3$$
$$23250 < q < 96722.4 \ W/m^2$$

where,

   Wav = Average circumferential velocity of flow on film surface, m/s.

   u = Average creep velocity of film flow m/s.

   q = Specific heat flow W/m^2.

To minimize the number of iteration steps in the calculation of film thickness according to Eq. (24) a first approximation for an estimate of average thickness of the film can be used (for the ratio of the velocities Wav/u). The simple equation:

$$\pi_\delta = 0.578 \ (Re_v)^{0.43} \tag{25}$$

is valid for $4.7 < Re_v < 1753.1$.

HEAT TRANSFER WITHOUT PHASE CHANGE

Two simplified theories have been advanced in the literature to predict the heat transfer coefficient. The first theory states that heat transfer through thin film is purely by steady state conduction. (Kern and Karakas 1959), Lustenader et al. 1959). The application of this theory is, however, limited because of experimental difficulties inherent in measuring the film thickness. The second theory proposes that the rate of heat transfer between the heating surface and the liquid layer is governed by unsteady state conduction in frequently renewed layer (Harriott 1959; Kool 1958 and Latinen 1959). On this basis the scraped film transfer coefficient is given by:

$$h_s = 1.13 \ \sqrt{\rho K C_p N B} \tag{26}$$

While the latter theory seems to predict with moderate accuracy the coefficients for water and liquids of low viscosity, the accuracy seems to decrease markedly when it is applied to conditions involving viscous materials (Harriott 1959). Kool (1958) extended this theory and derived an expression for the overall heat transfer coefficient assuming a model

for the fluid history. When the blade passes a point on the surface, complete mixing is postulated so that at that instant the temperature from the heat transfer wall to the processor axis is equalized. Again, the limitation of this theory is that it assumes the need for good mixing for heat transfer to viscous liquids and hence predicts higher coefficients than those obtained in practice.

Bott and Romero (1963) studied heat transfer to nonboiling water and water-glycerol mixtures inside a scraped surface, falling film vertical heat exchanger. The flow rates were 0.039-0.133 kg/s. Speed of rotation was 6.17-26.7 rps. and the number of blades ranged from 1 to 4. An approximate heat transfer correlation was proposed as follows:

$$Nu = 0.018 \ (Re_f)^{0.46} \ (Re_R)^{0.6} \ (Pr)^{0.87} \ (D/L)^{0.48} \ (B)^{0.24} \qquad (27)$$

The functional dependence of the heat transfer coefficient on various parameters is as follows:

$$h_s \approx (K)^{0.13} \ (C_p)^{0.87} \ (\mu)^{-0.19} \ (\rho)^{0.6} \ (D)^{0.22} \ (N)^{0.6} \ (\bar{M})^{0.46}$$

$$(L)^{-0.48} \ (B)^{0.24} \qquad (28)$$

The dependence of h_s on $K^{0.13}$ is questionable. One would normally expect that the exchanger length would not have such a pronounced effect on the heat transfer coefficient for it seems that high turbulence liquid in the film would destroy the stream line motion which is necessary for a pronounced dependence on length. Tests at a flow rate of 0.006 kg/s indicated "some anomalous behavior" and would not correlate with the other three flow rates (Min. of 0.012 kg/s); the 0.006 kg/s results were neither reported nor correlated. Another anomaly was also present i.e., the effect of rotation on the coefficient was greater for high axial flow rates than for low ones. It may be that these anomalies are due to the heat transfer surface becoming essentially dry behind the scraper for low flow rates.

Bott and Sheikh (1964) conducted tests, which included photographs of agitator operation, on the effectiveness of various dynamically loaded wiper designs (flat blade, in line gaps, staggered gaps, and holes in the edge) for heating water. The in-line gaps gave highest coefficients (about 20% higher than flat blades). They attributed this to the greater heating surface coverage of liquid for the inline gaps. With all wiper designs they apparently witnessed unheated patches behind the wiper; this led them to believe that heat transfer might be maximized by maximizing the wetted surface.

Bott and Azoory (1968) further investigated the heat transfer using a rotating knitmesh plug as the scraper element. As much as 20% to 30%

increase in heat transfer was observed over a similar exchanger using flat blades. Even directly comparing 'crimp blade' to flat blade systems, an increase of about 7.5% was observed. Further improvement could possibly be obtained with different knitmesh characteristics (i.e., stitches and crimps/m). The results indicated that for these elements, over the whole range studied (Pr = 15-500 for the glycerol solutions):

$$h_s \approx (N)^{0.3} \tag{29}$$

In the Eq. (27) the tube diameter has been used as the characteristic dimension. Bott and Romero (1966), and Bott (1966) pointed out at the time that this criterion may not be reliable when the tube diameter is substantially different from the one used in the experiments. It would be an improvement if the correlations were expressed in a form which would not change with variation in the diameter of the tube. The use of an equivalent diameter has been accepted in correlating heat transfer in complex situations, therefore, a modified form of the characteristic length was suggested, based on equivalent diameter assuming that liquid was carried in the form of triangular fillet in front of the blades. According to this model, the equivalent diameter is defined as $4r_h$, where the hydraulic radius r_h is the ratio of the flow area to the heated perimeter for triangular shape of fillet. Stoke's theorem for flow in an open channel makes it possible to use directly Boussinesq's expression for volume rate of flow through a vertical tube of rectangular cross section X. This consideration leads to the following expression:

$$X = \left\{ \frac{\dot{M}\mu}{0.0703 \, B\rho^2 g} \right\}^{1/2} \tag{30}$$

The equivalent diameter De = $4r_h$ = 4X/b, b = arc of the filler. b = $\sqrt{2X}$

$$De = \left\{ \frac{64 \, \dot{M}\mu}{0.0703 \, B\rho^2 g} \right\}^{1/4} \tag{31}$$

Based on equivalent diameter following correlation was developed:

$$Nu_{De} = 0.0023(Re_{ft})^{0.52} \, (Re_R)^{0.57} \, (Pr)^{1.0} \, (L/De)^{-0.44} \, B^{0.61} \tag{32}$$

The use of an equivalent diameter based on idealized model is intrinsically more sound that the use of tube diameter without regard to actual

conditions in the exchanger, and for this reason the correlation based on equivalent diameter is in a form more likely to be valid for extrapolation.

Rearranging Eq. (32) and splitting the dimensionless groups so that B and De can be written explicitly, it can be shown that

$$h_s \approx (D_e)^{-0.51} (B)^{0.09} \tag{33}$$

$$\text{But } D_e \approx (B)^{-0.25} \tag{34}$$

$$\text{therefore, } h_s \approx (B)^{0.22} \tag{35}$$

The exponent of 0.22 now obtained compares satisfactorily with 0.24 obtained in the earlier correlation with D as characteristic dimension.

Due to the complexity of the heat transfer mechanism and the added complication of free liquid surface that exists in thin film units, Bott and Romero approached the problem from an engineering point of view by considering the data collected over a wide range of operating conditions and establishing some correlations between the various factors involved. The above correlation is given in terms of fillet Reynolds number to account for axial flow rate, rotary Reynolds number for the effect of rotation, Prandtl number, ratio (L/De) and the number of blades. The selection of dimensionless groups, although carried out by dimensional analysis, may be regarded as somewhat arbitrary, and no distinction between various flow regimes is made.

Bott and Azoory (1969) attempted to derive the dimensionless parameters influencing the heat transfer mechanism in a scraped, thin film vertical thermal processor, from the governing differential equations and boundary conditions. Experimental work was carried out on two heat exchangers, using water and glycerol solutions (80%). The flow rate ranged from 0.013-0.038 kg/s to produce a uniformly wet surface. The scraper speed ranged from 5-27 rps. Uniform heat flux (electric heating) was assumed. The results were discussed in the light of the penetration model (Kool 1958). They considered the relative magnitude of all acting forces, viz. the viscous, centrifugal and gravitational forces. The modified general heat transfer correlations were as follows:

$$Nu_{De} = C(Re_{ft})^{0.58} (Fr_R)^{0.09} (Pr)^{0.57} \tag{36}$$

(For laminar flow regime $Re_{ft} < 100$).

$$Nu_{De} = C(Re_{ft})^{0.76} (Fr_R)^{0.36} (Pr)^{1.02} \tag{37}$$

For transition flow regime $100 < Rt_{ft} < 1000$).

The values of constant C depends upon geometry of the system (i.e., diameter, number and arrangement of the blades).

The influence of the axial velocity in thin film units may be regarded as greater than that in liquid full devices (Penny and Bell 1967) and the inclusion of a fillet Reynolds number based on the mean axial velocity becomes important. However, the presence of two Reynolds numbers in the final correlation may be superfluous because it was found from computer analysis of experimental results that the values of the exponents on the two Reynolds groups were interdependent. Also the range of the fillet Reynolds number in the above investigation was substantially larger than that of the rotary Reynolds number. As a result, the rotary Reynolds number obtained from the analysis has been replaced by a fillet Reynolds number which takes into account mean axial velocity. It was also desirable to have only one group containing the speed of rotation. The group chosen was Froude group $Fr_R(N^2D^2/De\ g)$ which represents the ratio of intertial (in this case centrifugal) to gravitational forces. The inclusion of Froude group becomes significant in thin film operation due to the presence of a free liquid surface which has been ignored by previous workers.

The ratio (average of all data) of theoretical heat transfer coefficient based on penetration model to the experimental values was given as:

$$\frac{h_s\ (\text{theoretical})}{h_s\ (\text{exp.})} = 1.212\ (\text{for laminar regime}) \tag{38}$$

$$\frac{h_s\ (\text{theoretical})}{h_s\ (\text{exp.})} = 0.807\ (\text{for transition regime}) \tag{39}$$

In the transition region, the average experimental value of the heat transfer coefficient was approximately one and a quarter times greater than those predicted from penetration model. The flow rate which is not a variable in penetratic.. model, improves the rate of heat transfer and therefore, accounts for the higher values of the experimentally determined h_s from those predicted by the model. The functional dependence of the experimental value of the coefficient on the other parameters is given below:

$$h_s \approx (\mu)^{0.11}\ (N)^{0.56}\ (K)^{-0.02} \tag{40}$$

A slightly greater dependence of h_s (exp) on N can be seen as compared to $N^{0.5}$ postulated by ideal penetration model but, nevertheless, the two are in good agreement.

In the laminar region, the average value of the experimental heat transfer coefficient was approximately 0.8 times the predicted average value. The

application of Kool's model in this region is rather limited because it assumes the need for good mixing. The functional dependence of h_s on speed of rotation, liquid thermal conductivity and viscosity is:

$$h_s \approx (N)^{0.18} (K)^{0.43} (\mu)^{0.13} \qquad (41)$$

The dependence of the coefficient on viscosity in the region is relatively small. This confirms, within the experimental error allowed in this analysis, that the heat transfer coefficient is independent of liquid viscosity in the laminar region. The greater dependence of h_s on K might indicate that the heat transfer mechanism is more dependent on heat conduction through the stagnant film in the liquid fillet adjacent to the wall. Further more, from these limited results in an exchanger with relatively high length to diameter ratio, h_s is proportional to $N^{0.18}$ and not $N^{0.5}$ as the penetration model predicts.

The above analysis reveals the inadequacy of present theoretical models of predicting the heat transfer coefficient in thin film units. Until such time as modification of penetration model is made to take into account the effect of flow rate and degree of mixing, the use of empirical correlations if properly derived from basic flow and heat equations, will be adequate for general design and scale up.

Azoory and Bott (1970) tried to relate the time averaged local heat transfer coefficient (hsl) as measured by a plug-type flux meter, to the penetration model. From their experimental measurements, they developed an empirical correction factor which was found to be a function of a Prandtl number, to make their results agree with the model formulation, viz:

$$h_s l = 1.128 \, (\rho C_p K \, N \, B)^{\frac{1}{2}} (1/f); \text{ (S.I. System)} \qquad (42)$$

where $f = \dfrac{Pr}{500} + 3.5$

For Prandtl number (Pr) in the range 7-360 and Re_{ft} between 100 and 100,000.

The main conclusions were (1) there is good agreement with the penetration model that the local heat transfer coefficient is a function of $N^{0.5}$, (2) it is possible to describe the heat transfer by applying a correction factor to the theoretical penetration model. The correction term appears to be independent of flow rate and rotational speed but dependent upon the liquid physical properties and (3) in these experiments the local scraped surface heat transfer coefficient is independent of flow rate as predicted by the penetration model.

Bott and Nair (1969) studied the behavior of glycerol water mixtures of higher viscosity than hither to and two nonNewtonian Pseudo-plastic liquids with the purpose to extend the range of studies to high Prandtl numbers and to an assumed laminar flow regime of $Re_{ft} < 100$. Solutions of 1.5% sodium carboxy-methyl-cellulose and 1.3% polyethylene oxide in water were used as being representative pseudo-plastic materials. It was found that the experimental results could be correlated by an equation similar in form to Eq. (42). The correction factor f was again found to be independent of liquid flow rate but dependent upon fluid properties included in the Prandtl number.

For the assumed laminar regime of $Re_{ft} < 100$

$$f = 2.3 \ (Pr)^{0.13} \tag{44}$$

For Prandtl number in the range 670-2200.

Miyashita and Hoffman (1978) determined instantaneous heat transfer coefficients using electro-chemical technique. The time averaged local heat transfer coefficients (hsl) agreed fairly well with those of Azoory and Bott (1970), however, some variation with liquid flow rate was observed. The time averaged local Nusselt number (Nul) for two bladed scraped surface heat exchanger was found to be represented by

$$Nul = 0.15 \ (Re_R)^{\frac{1}{2}} \ (Pr)^{\frac{1}{2}} \ (Re_f) \cdot a \tag{45}$$

$$\text{where } a = \frac{1 \ - \ 0.038 \ N}{9}$$

The variation in the instantaneous coefficient with time was very different from that penetration model based on unsteady state heat conduction.

Skoczylas (1978) developed similar dimensionless correlations for heat transfer in liquid film in "Sambay" type heat exchangers. The liquids used were ethylene glycol, glycerol, aq. glycerol and sugar solution. The advantages of "Sambay" type heat exchangers over static type thin film heat exchanger were shown by comparing the heat transfer coefficients for both cases.

HEAT TRANSFER WITH PHASE CHANGE (EVAPORATION)

Two different mechanisms of heat transfer occur for a flowing, boiling and mechanically affected film. According to first mechanism, heat transfer

takes place by conduction across a vapor film combined with evaporation of more volatile component at the surface of the film. This mechanism can be expected at low values of the specific heat flux and with laminar flow of the film. The flow of the film can be regarded in such a case as the resultant of the internal friction, the force of gravity and the peripheral forces created by the effect of the rotation (Kramers *et al.* 1955).

According to the second mechanism heat transfer takes place due to vapor bubbles formed at heat transfer surface or at overheated spots inside the film. This mechanism can be expected with higher values of specific heat flux and with turbulent flow of film. The flow of the film may be regarded as the resultant of the forces of internal friction, the gravitational force and the peripheral force affected also by the formation and release of vapor bubbles (Ziolkowski and Skoczylas 1965).

The majority of work was directed towards generating data for specific systems rather than developing general design methods. Practical operating data for the Sambay (dynamically loaded wiper) and luwa (fixed clearance wiper) evaporators were given by Dieter (1960), Krischbaum and Dieter (1958), Leniger and Veldstra (1959) and Schneider (1955). In general, they found that temperature difference (ΔT) between liquid and heating medium and the axial flow rate of liquid had little effect on heat transfer coefficient. Krischbaum and Dieter however found that for low axial flow rates and large temperature differences, the coefficient decreased with an increase in ΔT at constant rotational speed.

Bressler (1958) suggested that there was an optimum rotor speed beyond which an increase in speed had little effect upon heat transfer. In contrast, Krischbaum and Dieter presented data in the following correlation which shows no such limitation:

$$h_s = 437 \, \frac{N^{0.33} P_K}{\mu} \, ; \, W/m^2 \, K \qquad (46)$$

Limited operating data on similar devices were given by Borg *et al.* (1955). Lustenader *et al.* (1959) investigated the evaporation of water in a thin film unit with various spring loaded wipers. Water wettability was found to be most important wiper design criterion because nonwettable wipers eventually lay down a hydrophobic film on the heat transfer surface. The experimental data showed that heat transfer coefficient experienced a maximum as ΔT or shaft speed(N) increased with the other held constant. A theory which had its basic consideration of the liquid film as a pure resistance was developed which fitted the experimental data very well. The theory cannot be used a priori, for no method is given for pre-

dicting the clearance (and therefore the initial film thickness); the clearance (in the range of 0.0013×10^{-2} m) was back calculated from the experimental data.

Charles (1959) experimented with a heat exchanger having plane heat exchange surfaces and provided with continuous mechanical scrapers. High (unspecified) heat transfer coefficients were claimed. The equipment gave constant production rates without shut downs for cleaning and required low energy input. The heat exchanger was used for sea water evaporation with water evaporation capacity ranging from 0.061 to 14 kg/s.

Per Hilmo (1961) in a specially constructed experimental model of a vertical thin film evaporator, studied the effect of rotor on fluid flow and heat transfer during evaporation. Results indicated that a liquid wave of relatively large crosssectional area is formed in front of the rotor blades and that a significant portion of evaporation takes place in this wave. In the evaporation of water at atmospheric pressure, the heat transfer coefficient increased as the average heat flow was increased. This may be due to formation of steam bubbles in the bulk liquid or on the film surface, either mechanism would reduce super-heating of the liquid. The overall heat transfer coefficient decreased when the vacuum was increased, because viscosity increased and a greater portion of heat transfer surface was contacted by the vpaor. At the lowest evaporation pressure, a burn out phenomenon appeared. This might occur periodically as the liquid is brought in contact with the wall of the apparatus each time one of the rotor blades passes by. In evaporating aq. glycerol solutions, the heat transfer coefficient first increased slightly as the glycerol concentration was raised (from 0 to 4%) and then decreased steadily with progressive increase in viscosity.

Reay (1963) studied the relationship between rotor clearance and thermal performance of a vertical thin film evaporator. Various fixed clearance rotors were used. The results showed that as the blade tip clearance is reduced, the overall heat transfer coefficient based on the total evaporator surface, increased. If the clearance is doubled, performance drops only 20%, and if the clearance is decreased by 80%, performance is increased by approximately the same 20%. When blade tip clearance is reduced to a certain value, higher liquid loading does not cause any change in heat transfer coefficient, once the whole surface becomes wet.

Robert et al. (1963) studied the details of a thin film evaporation in an outside wiped film (O.W.F.) evaporator. A system of rotating wiper blades and feed tubes produced a continuous liquid film on the 0.1 m diameter outer surface of a vertical copper tube, 0.4 m long, heated internally by condensing steam. The principal independent variables were temperature difference across the film (ΔT), blade speed (N), evaporation pressure

and feed flow rate, salinity and its temperature. An average wiped film thickness under nonevaporating conditions (δ), was measured by suddenly stopping the blade rotation, blotting the liquid from the tube surface and accurately weighing the moist blotter. It was found that δ increased with feed flow rate and varied as square root of N i.e. ($\delta \alpha \sqrt{N}$), as predicted by laminar slider lubrication theory. This stop and sop method was used to adjust the loading of all the blades to produce a uniform film thickness. In evaporation trials validated by mass and heat balances, experimental evaporation rates and heat transfer coefficients were correlated with photographic observations of evaporation surface with theory. Low blade speeds and feed flow rates gave rise to dry wall areas in the circumferential and axial directions, respectively. The evaporation rate increased with ΔT and evaporator pressure, but then decreased with increased salinity.

Robert et al. (1964) conducted evaporation experiments with wiped film and falling saline water film (without wiping). The same heat exchanger as described above, was used. The tube was surrounded by a glass pipe which permitted visual observation. A 0.3 to 1.0 mil thin wiped film was obtained from three flexible rubber blades attached to a cage rotating at 1 to 2 rps. The blades were fed at the top by corotating feed ducts and feed flow rate was constant at 0.0108 kg/s. Six rotating feed ducts supplied a falling film in Reynolds number range of 160-600. Temperature difference across the film ranged up to 10 °C for falling and 18 °C for wiped films. Film stability was poor for pure water but high for saline water. Overall evaporative heat transfer coefficients were in the range of 4800-5700 W/m² K for falling film and 11,000-17,000 W/m² for wiped films. Performance comparison in terms of specific evaporation rate (kg/s m²) versus overall pressure difference showed that at low feed, salinity or evaporation rate the specific yield with wiped film was greater by a factor of 2.3 of that with falling film. At high feed, salinity or evaporation rate, specific yields were nearly the same.

Bott and Sheikh (1966) investigated heat transfer to boiling water and boiling aq. glycerol in a specially designed laboratory scraped surface vertical heat exchanger (3.75 × 10⁻²m diameter, 45 × 10⁻²m long tube). Evaporation took place on the inner wall under the action of up to eight plain floating blades made from laminated phenolic plastic. Heat was supplied by 6 kW electric windings around the outside of the tube and suitably controlled.

Evaporation of water demonstrated that, over the range of variables studied, at lower speeds (10-15 rps.), h_s is not altogether independent of ΔT, implying that at these shaft speeds some slight nucleation or bubble formation at the surface is taking place. At higher speeds (20-23 rps.). h_s is more or less independent of ΔT. Under such conditions, it is con-

sidered that there is little bubble formation at the surface. Super-heated liquid is swept from the transfer surface by the action of the blades and vaporization is by flashing at the free liquid surface remote from transfer surface. In addition, evaporation heat transfer appeared to be independent of film Reynolds number. The scraped surface coefficient for boiling water increases with speed of rotation in accordance with the following correlation:

$$h_s \approx N^{0.37} \tag{47}$$

which is in good agreement with that by Dieter and Krischbaum who demonstrated the dependence of the coefficient on $N^{0.33}$: Heat transfer to boiling glycerol solutions has been shown to be correlated by the following equation (Prandtl number range 5-61).

$$Nu = 0.65(Re_f)^{0.25} \ (Re_R)^{0.43} \ (Pr)^{0.3} \ (B)^{0.33} \tag{48}$$

The groups investigated do not involve properties associated with bubble formation, since it is again considered that the action of blades prevents extensive bubble formation. In spite of this, the relationship does not bear a marked similarity to the correlation previously proposed for heat transfer without change in phase (Eq. 27), except as for as the exponent on B is concerned.

The exponent 0.43 on rotating Reynold's number is in reasonable agreement with exponent of 0.37 on speed of rotation in the experiments with water and the results of Krischbaum and Dieter.

The low value of the exponent on the film Reynolds number shows that in the range below 1000, this group has some influence on heat transfer. Even with water at $Re_f = 1000$, there is indication that heat transfer is a function of film Reynolds number. This is in contrast to majority of the results for water ($Re_f > 1000$), particularly at higher shaft speeds, where heat transfer is independent of film Reynolds number.

The exponent on the Prandtl group of 0.3 is similar to that generally accepted for turbulent flow in pipes.

Skoczylas (1970) conducted about 265 experiments to study boiling heat transfer coefficients in case of distilled water, toulene, methanol and ethylglycol solutions. A laboratory Sambay type evaporator consisted of copper tube of 7.9×10^{-2} m diameter and 85.0×10^{-2} m length. The heating was done through electric heaters with continuous regulation from 0 to 33 KW. Thirty blades were fitted on the shaft length of 85.0×10^{-2} m with three blades at each level (120° apart) and 0.5×10^{-2} m blade overlap. Specific heat was introduced as an independent variable as the

liquid gets super-heated during the boiling process. Heat transfer coeffi-
cient was assumed to be independent of the temperature difference bet-
ween the tube wall and boiling liquid (ΔT) and specific gravity of vapors.
Following generalized correlation was obtained.

$$Nu = 3103(Re_f)^{-0.988} \ (Re_R)^{0.404} \ (Pr)^{-1.053} \ (B)^{-0.326} \ (We)^{0.494} \qquad (49)$$

In order to check the usefulness of the correlation for calculating heat
transfer over a wide range of physico-chemical properties, additional tests
were conducted on boiling ethylglycol at 30 mm Hg pressure. Few ex-
perimental values were found to be considerably higher than calculated
values. This was attributed to carry over of boiling liquid by the vapors
when the vapor speed exceeds the permissible value.

The presence of Re_R in Eq. (49) seems to be superfluous as the effect
of rotational speed is depicted in Weber number 'We' also.

Gudheim and Donovan (1957) compared the thermal performance of
vertical straight- sided with horizontal reverse-tapered thin film centrifugal
processing units. Both the units had a wall thickness of 0.25×10^{-2} m.
It was observed that the product side heat transfer coefficients in horizon-
tal machine were extremely high at least as high as steam film coefficient.
The principal resistance to heat flow in this case is in the metal wall itself.
When the horizontal unit receives sufficient feed to permit absorption of
all available heat, all the coefficients obtained are at a maximum value
which is a constant. Here the processing fluid heat pick-up is limited by
the maximum heat transfer rate of which the unit is capable. The straight
sided vertical unit does not reach this critical feed rate except at values
so high that recycling is probably necessary.

The effects of varying temperature differences (ΔT) and vacuum were
studied in both types of heat exchangers. It was shown that within the con-
fines of tests (flow rate: 0,034-0.085 kg/s m²; T: 10-40°C and vacuum:
0-710 mm Hg), the coefficients were the same in horizontal unit, irrespec-
tive of the vacuum temperature differences or feed rate. However, varia-
tions in latter factors had a significant influence on heat transfer in the
vertical machine.

An independent influence of viscosity was appraised and for this pur-
pose, guar-gum was selected as test fluid. The examination of data revealed
that the overall heat transfer coefficients decreased only moderately (from
3400 to 3100 W/m²K) but uniformly with increasing viscosities up to an
actual operating bulk temperature viscosity of about 10 Pas. Then the values
began to decrease at a much faster rate to a U value of 1980 W/m²K at
40 Pas since U values of 3975 W/m²K were obtained with pure water,
this represents a 50% decrease in heat transfer rate. However, the resistance

to heat transfer due to viscosity was less in horizontal tapered unit than in straight sided vertical unit.

Chaing (1976) obtained experimental heat transfer data with water and a mixture of $CHCI:CCI_2$ 95% and mineral oil 5% in a horizontal, wiped film evaporator and obtained a correlation as:

$$Nu = 1.5(Pr)^{-0.42} (Re_f)^{1.6} (L/D)^{-2.2} (W/Tc)^{-1.4} f(E) \qquad (50)$$

where, $f(E) = 1 + 0.0242(E) + 0.00817(E)^2 - 0.000046(E)^3 \qquad (51)$

W - Weighted log mean temperature difference.
Tc - Critical boiling temperature difference.

CONCLUSIONS

From the foregoing discussion the following inferences could be drawn: (1) The thin film operation is complex from a predictive stand point. The liquid film thickness varies circumferentially and in some cases axially. The heat transfer surface can become dry, specially during evaporation. In view of these anomalies it is not appropriate to use the empirical correlations of dimensionless groups, represented by the equations, outside the range of data for which they are developed. (2) For flows in complex units like scraped surface heat exchangers, experimental determination of the residence time distribution is necessary because the flow patterns for these instances become too complicated to be adequately described by a mathematical model. (3) It has been reported that power input to the rotor is mainly consumed in internal friction and turbulence within the liquid. Therefore, it is essentially a function of viscosity. But in thin film operation where a new fluid surface is generated mechanically, the effect of surface tension forces cannot be ignored. Hence, the dependence of power consumption on surface tension forces needs investigation. (4) Most of the results so far have been obtained using 2.5×10^{-2} m to 10×10^{-2} m diameter and 0.5 to 1 m long vertical wiped/scraped surface heat exchangers. The heating has been through electrical energy. However, in order to get insight of actual conditions, investigations on larger exchanger with steam as heating medium need to be done. (5) Within the confines of tests, the thermal performance of horizontal reverse-tapered thin film centrifugal processing unit is far superior to that of vertical straight sided unit. (6) The effect of weight of blades on scraped film heat transfer coefficient has not been studied. (7) Practically no design correlations as regards to hydrodynamics and heat transfer in straight sided horizontal thin film scraped surface heat exchangers are available.

NOTATIONS

B	= Number of blades.
C_p	= Specific heat.
D	= Inside diameter of heat exchanger.
D_e	= Equivalent diameter.
F_{av}	= Average amount of liquid at boiling related to unit length of circumference.
g	= Acceleration due to gravity.
g_c	= Newton's Law Conversion factor.
h_s	= Scraped film heat transfer coefficient.
K	= Thermal conductivity of fluid.
L	= Length of heat exchanger/rotor.
$\dot{M}$	= Mass flow rate.
N	= Rotor speed.
V_c	= Circumferential velocity of rotor blades.

Greek Letters

γ	= Shear rate.
δ	= Film thickness.
μ	= Coefficient of viscosity.
ρ	= Density.
σ	= Surface tension.
τ	= Shear stress.

Dimensionless Groups

Fr $\quad$ = Froude number, $\dfrac{N^2 D^2}{\delta g}$

$Fr_R \quad = \dfrac{N^2 D^2}{De\ g}$

Fv $\quad$ = Criterian for physical properties, $\dfrac{\sigma^3 \rho}{g \mu^4}$

Nu $\quad$ = Nusselt number, $\dfrac{h_s D}{K}$

Pr $= $ Prandtl number, $\dfrac{\mu C_p}{K}$

Re_f $= $ Film Reynolds number, $\dfrac{\dot{M}}{\pi D \mu}$

Re_{ft} $= $ Fillet Reynolds number, $\dfrac{8\,\dot{M}}{B\,De\,\mu}$

Re_R $= $ Rotational Reynolds number, $\dfrac{D^2\,N\rho}{\mu}$

Re_R $= \dfrac{D\,De\,N\rho}{\mu}$

Re_v $= $ Reynolds number for flow at boiling point $\dfrac{4F_{av}}{\mu}$

We $= $ Weber number, $\dfrac{\rho\,N^2\,D^3}{\sigma\,g_c}$

π_δ $= $ Criterion for film thickness $\dfrac{\delta}{\left\{ \dfrac{1}{g}\left(\dfrac{\mu}{\rho}\right)^2 \right\}^{1/3}}$

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Tables should be numbered consecutively with Arabic numerals. The title of the table should appear as below:

Table 1. Activity of potato acyl-hydrolases on neutral lipids, galactolipids, and phospholipids

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CONTENTS